Financial Literacy, Information Acquisition and Asset Pricing Implications

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In this paper I study the information acquisition process in a simple asset pricing model with heterogeneous beliefs about future prices. This is instrumental to investigate the effects of financial literacy on market volatility. I posit that financial literacy affects the cost of acquiring information on the asset payoff and show that the effect on the market volatility is cross-partial non-monotone and depends on the uncertainty of the fundamentals. I conclude that policies aimed at reducing the financial information acquisition cost increase aggregate informativeness and, in a scenario with high uncertainty of the fundamentals, reduce the market volatility. The main intuition is that lower information cost for the less literate households leads them to acquire more private information and to trade more actively. Having more private information revealed by the market price affects positively market volatility. On the other hand, with low uncertainty in the fundamental, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively. Moreover, reducing the information acquisition cost is not welfare improving if the policy makers introduce a proportional tax on the excess return to finance the policy expenditures.

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1 Introduction

Due to the changes occurred in the welfare state and in the ageing path of the industrialized countries, the burden and the risk of the financial choices shifted onto the individuals. Thus, the role played by the financial literacy in the financial decision process of an investor captures the attention of the policy makers.\footnote{I use the following definition of financial literacy: degree of knowledge of basic financial concept, ability to manage personal finances, and confidence of own choices made in a complex financial environment. For a review of financial literacy definitions Remund (2010), Hung et al. (2009).}

Lack of abilities or skills to manage lifetime financial wealth, such as misunderstanding of financial matters or underestimating current opportunities and future needs, becomes a crucial issue in the policy makers' agenda, especially during the times of crisis. Identifying the "vulnerable population" and providing financial education are the two main policies implemented by the financial regulators.\footnote{According to the definition provided by OECD-INFE, the "vulnerable populations" include groups likely to face economic challenges and less likely to fully participate in the financial mainstream. These groups may include, but are not limited to: youth and young adults, the unemployed and under-employed, low-income consumers, those with little or no savings, other consumers outside of or partially outside of the workforce (for example, people with disabilities), and other vulnerable sociodemographic groups such as women and racial and ethnic minorities. See OECD website for a list of financial education programs: http://www.financial-education.org/home.html} However, it is still under debate the effects of these policies, especially the latter. While from the theoretical point of view, partial equilibrium models provide rationale for financial education programs (Jappelli and Padula, 2011), the empirical literature shows mixed evidences about the individual benefits (Lusardi and Mitchell (2009), Christelis et al. (2010), Lusardi and Tufano (2009)). In order to explain non-optimal financial performance of the households, such as low equity market participation or low level of portfolio diversification, a stream of household finance literature focus on the individuals abilities to process information: cost of acquiring information (Peress (2005), Nieuwerburgh and Veldkamp (2010)), overconfidence (Odean, 1998), awareness (Guiso and Jappelli, 2005), limited cognitive abilities ((Christelis et al., 2010)).

I follow the framework based on the strategic substitutability in the information acquisition to analyze the effect of policies aimed at improving the individual financial abilities, such as reducing the information acquisition cost, i.e. implementing better transparency rules for the financial prospects or increasing the average degree of financial literacy in the population through financial education programs.\footnote{I model explicitly how agents acquire information and how financial literacy affects this process. My paper follows the literature on information acquisition with costly precision.}

Structural models are necessary to capture the feed-back effects of such policies on the individual behaviour, in particular the informational exter-
nalities, and to derive asset pricing implications. My approach takes into account the feedback effects within a general equilibrium framework. I investigate the effect of a reduction in the information acquisition cost for the less literate agents on the volatility of the market. As a proxy, I use market price variance. The impact of the policy on the market volatility is cross‐partial non‐monotone and depends on the uncertainty of the market fundamentals. Even if the aggregate informativeness always increases, the market volatility decreases with high uncertainty in the fundamentals and it increases with low uncertainty. The main intuition is that lower information cost for the less literate households leads them to acquire more private information and to trade more actively. Therefore, more private information revealed by the market price affects positively market volatility. On the other hand, with low uncertainty in the fundamentals, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively.

Similar CARA-Gaussian models were used to explain inequalities among households, e.g. Verrecchia (1982) through heterogeneous risk aversion and Peress (2004) through heterogeneous initial wealth. The main implication is that wealthier households gain more from purchasing the private information, improving their Sharpe ratio. Thus, they end up to be more informed. My model differs in the source of heterogeneity. I consider heterogeneous information acquisition costs. The households can purchase an unbiased additive noisy signal on the payoff of the risky asset. They can choose the amount of the signal precision and what they pay is proportional to their financial literacy. The policy makers are able to manipulate the degree of heterogeneity of individuals through financial policies such as transparency improving rules or financial education programs. Similar policies are not easily implementable if we take into account heterogeneity in risk aversion or in other subjective characteristics.

The results show that more financially literate agents are, more information they purchase and more revealing market prices are. The impact on market volatility is cross‐partial non‐monotone and depends on the fundamentals uncertainty. I conclude that the policy makers can reduce the volatility of the market only in a scenario with high uncertainty, namely when the financial information is more valuable. Conversely, with low uncertainty, improving the financial market transparency leads to higher volatility. Using the model, I perform a policy exercise letting the policy makers provide financial education programs. Reducing the information acquisition cost is not welfare improving if the policy makers introduce a proportional tax on

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4I figure out a situation where agents face the same financial report and extract signals on the true payoff paying a cost. More expert they are, lower costs they have, more precise signals will be. Their ability to understand financial information is exogenous. Padula and Pettinicihi (2012) take it endogenous, letting the agents optimally choose the amount of financial literacy they want to accumulate.
the excess return to finance the policy expenditures.

The paper is organized as follows: in Section 2 I set up the model, in Section 3 I characterize the equilibrium and I discuss the implications of the model. Section 4 performs the welfare analysis and the last section concludes and points out further research steps. All the proofs are collected in the Appendixes.

2 Model

In this model, the agents face two choices: in the first period, they have to choose if and how much information they want to purchase. In the second period, if and how to trade in the market. There are two primitive assets available for trading. A riskless asset, that is perfect elastically supplied, pays a rate of return \( R = 1 + r \). A risky asset, with price \( p \), pays a payoff \( \pi \) with \( \pi \sim N(\mu_\pi, \tau_\pi^{-1}) \). Short selling is allowed and a tax rate \( t \) affects the capital gains of the traders.

The per capita supply of the risky asset is \( \theta \sim N(\mu_\theta, \tau_\theta^{-1}) \), i.e. noise trading.\(^5\) The fundamentals, \( \pi \) and \( \theta \), are mutually independent random variables and their joint distribution is common knowledge.

Agents and policy makers

Agents differ in their ability to acquire information. Heterogeneity is expressed by \( c \), that affects the costly information acquisition process. Without loss of generality, I assume two types of agents. Type \( L \) (literate agents) with low cost and type \( H \) (illiterate agents) with high cost of acquiring financial information. \( J = L \cup H \) is the set of all the agents. Both types of agents maximize the same concave utility function of their final wealth. For tractability, I assume CARA utility function with absolute risk aversion coefficient \( \rho \): \( U(W) = -\frac{1}{\rho} e^{-\rho W} \).\(^7\) Population of agents has mass one and, for

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\(^5\) The assumption of unbounded normally distributed payoffs allows to work out a closed form solution. Alternative distributional assumptions on payoff shocks are used by Barlevy and Veronesi (2007) and Breon-Drish (2012).

\(^6\) The introduction of an exogenous aggregate shock allows to avoid the Grossman-Stiglitz paradox. With an extra noise, market price are not fully revealing, therefore agents still have some incentives to purchase information. In the literature, different interpretations characterize this assumption: presence of irrational noise traders, random endowment shocks (Biais et al., 2010) or other types of individual shocks such as liquidity needs (Wang, 1994).

\(^7\) The assumed shape of the utility function fully satisfies the participation principle: given a positive equity premium, all agents invest money in the risky asset, regardless of the degree of risk aversion or the riskiness of the asset. Empirical studies show that the participation principle fails in reality (the stock-holding puzzle): limited market participation and heterogeneous portfolio behaviours characterize real financial markets (Haliassos and Bertaut (1995), Guiso et al. (2003)).
each type, there are enough agents so that the law of large numbers applies.\footnote{Within the group agents differ only for the realization of their private signal, if they purchase one. Moreover, I posit that they do not realize they can act strategically, affecting market price through their informative choice and their asset demand.}

The participation in a financial education program provided by the policy makers motivates the differences between the two types of agents. Those agents who participate in the program become more literate and reduce their costs of acquiring financial information. However, the policy makers face a monetary cost to provide the financial education program. It is proportional to the fraction of agents who participate in. I denote with $\lambda$ the fraction of literate agents in the model. The policy makers are budget constrained and they must finance the expenditures of the program with the tax revenue from the trading. The policy makers optimally set the tax rate $t$, in order to satisfy the budget constraint such that the cost of the policy equals the expected fiscal revenue.

### Information structure

Once $\pi$ is realized but not revealed, each agent can purchase an unbiased signal $s$ about the risky asset payoff:

$$s = \begin{cases} \pi + \sqrt{\frac{1}{x}}\epsilon & \text{if } x > 0 \\ \emptyset & \text{if } x = 0 \end{cases}$$

where $\epsilon$ is a white noise, independent of $\pi$, $\theta$, and across agents. Agents can purchase private signal precision $x$ paying an opportunity cost $C(x,c)$.

Formally, the cost of acquiring an amount $x$ of precision is modelled by a continuous and twice differentiable function $C(x,c)$ over $x \in \mathbb{R}^+$ that is strictly convex: $C_x > 0$, $C_{xx} \geq 0$ and $C_c > 0$, $C_{cc} \geq 0$. It is continuous at $x = 0$: $C(0,c) = 0$, $\forall c$. And $\lim_{x \to +\infty} C(x,c) = +\infty$. The two properties imply that a totally uninformative signal is costless and a fully revealing signal is infinitely expensive. Moreover, the marginal cost $C_x$ is increasing in $c$ ($C_{xc} > 0$): acquiring information at the margin is more costly for less literate agents. These assumptions ensure the existence of a solution for the information choice. To illustrate the main intuition, I provide a simplified example that I will use to derive the numerical outcomes.

#### Example (Cost Function)

The cost function is: $C(x,c) = c(x^2 + x)$ with $c = \{c_L, c_H\}$ and $c_L < c_H$.

To solve the model, I focus on a partially revealing noisy rational expectation equilibrium. All agents have rational expectations in the sense of Hellwig (1980). After having privately observed unbiased signals, the agents transfer their private information to the market price through their asset demand. Then, the market price reveals some information about the true value
of the risky asset and the agents use it as an informative signal. However, the market price is not fully revealing given the noisy trading. Following the literature, I look for an equilibrium in which the market price is a linear function of the fundamentals: \( pR = a + b\pi - d\theta \) where the coefficients \( a, b, d \) are determined in equilibrium, imposing fully rationality of the agents.\(^9\)

I denote the agent \( j \)'s information set as \( \mathcal{F}_j = \{s_j, p\} \) where \( s_j \) denotes the agent \( j \)'s private signal and it is informative only if the agent \( j \) acquires some information precision.

### Timing

There are two periods. In period 1, the planning period, the agent can purchase a private signal \( s \), choosing its precision \( x \). In period 2, the trading period, after having observed her private signal realization \( s \) and the market price \( p \), the agent trades in a competitive market, choosing her portfolio share \( \alpha \). Figure 1 provides the timeline of the model.

![Timeline](image)

Figure 1: Timeline

### 3 Equilibrium

I solve the model by backward induction. I compute the optimal choices of the agents. Then, I compute the tax rate that satisfies the budget constraint of the policy makers.

In the trading period, the agent faces a portfolio allocation problem where she needs to choose the share of the portfolio invested in the risky asset, in order to maximize her expected utility. At this point in time, the precision \( x \) is already purchased and the initial welfare is reduced by the amount \( C \) spent for acquiring the private signal. The agent observes a private and a public signal (the market price) and computes the posterior beliefs about the asset payoff: \( E[\pi|\mathcal{F}] \) and \( Var[\pi|\mathcal{F}] \).

In the planning period, the agent chooses how much private signal precision \( x \) she wants and pays the monetary cost \( C(x, c) \), that is affected by

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\(^9\)Linearity is a standard assumption in the literature when the aim of the research is to find a closed form solution for the price function. Working with non-linear price functions provides a more general approach but loses the tractability of the solution.
the type specific parameter $c \in \{c_L, c_H\}$, the ability of acquiring financial information.

I describe the individual problem in each period and I provide the definition of an equilibrium within the family of the noisy rational expectation equilibria. Proposition 1 claims the existence and the uniqueness of the equilibrium. In the last part of the section 3, I discuss the results of the model.

The trading period

In this period, each agent maximizes the utility of the final wealth, optimally allocating the financial portfolio:

$$\max_{\alpha} E[U(W_2)|F]$$

subject to the budget constraint:

$$W_2 = (W_1 - C)R^p$$

where $R^p$ is the portfolio return:

$$R^p = \alpha(\pi - pR)(1 - t) + R$$

The tax rate affects only the capital gains of the risky asset and the sunk cost $C$ is given by the information choice made in the previous period.

The optimal share invested in risky assets differs between agents, depending on the signal observed and the precision purchased. In the trading period, all the information choices are already done and each trading agent transfers some of the information purchased to the market price through the risky asset demand. So, the market price partially reveals the private information. Therefore, when the rational agents form their posterior beliefs and formulate their asset demands, they take into account the aggregate informativeness and transform the market price into an unbiased public signal.

The indirect utility for agent $j$’s portfolio problem is $E[U(W_2^j)|F_j]$, which I note as $v(s_j, p; \Theta)$, where $\Theta$ is $\{c_L, c_H, R, W_1, \rho, \mu_\pi, \mu_\theta, \tau_\pi, \tau_\theta, \lambda, t\}$.

The planning period

In the planning period, each agent maximizes the indirect utility with respect to the information choice, the precision of the private signal. To simplify the notation, I drop the subscript $j$ and write:

$$\max_{x \geq 0} E[v(s, p; \Theta)]$$

subject to:

$$W_1 \geq C(x, c)$$
where the expected utility is computed over the joint probability distribution of $s$ and $p$. Recall that signal precision $x$ affects, by assumption, only the distribution of the private signal $s$. The individual optimal precision $x^*(\Theta)$ depends directly on the same type specific information acquisition cost and indirectly on the other type specific information acquisition cost through the aggregate informativeness. In the information market, it must hold the equilibrium condition, such that the aggregate informativeness is given by all the private information optimally acquired.

**The equilibrium**

A rational expectations equilibrium is given by an asset demand function $\alpha_j$ and an information demand function $x_j$ for all the agents, a price function $p$ of $\pi$ and $\theta$, and a scalar $I$ such that:

1. $x_j = x_j^*(\Theta)$ and $\alpha_j = \alpha^*(s_j, p; \Theta)$ solve the maximization problems.

2. $p$ clears the market for the risky asset:

   $$\int_{j \in J} \alpha_j \frac{W_1 - C_j}{p} dj = \theta$$

3. The informativeness of the price $I$, implied by aggregating individual precision choices, equals to the level assumed in the agents' maximization problem:

   $$I = \int_{j \in J} x_j dj$$

In noisy rational expectations equilibrium models, the agents make self-fulfilling conjectures about prices and the equilibrium is defined as the set of allocations such that the agents maximize their utilities, the markets clear and the individual optimal choices are consistent with the aggregate variables.

The following proposition claims the existence and the uniqueness of an equilibrium within the family of noisy rational expectation equilibria.

**Proposition 1.** There exists a unique noisy rational expectation equilibrium with linear price function.

**Proof.** See the appendix. \(\square\)

Here I provide a sketch of the proof and hereafter I discuss the main features of the equilibrium. For given aggregate informativeness $I$, I derive the market equilibrium, computing the optimal asset demands and the equilibrium market price function. Then, I compute the optimal information choices, checking when these individual choices are consistent with the assumed aggregate informativeness.
The information choice

The optimal information choice $x^*$ is the maximum between zero and the solution of the following equation:

$$2\rho R C_x(x^*, c)k^* = 1$$  \hspace{1cm} (3.1)

where $k$ is the precision of the individual posterior beliefs after having observed the private signal and the market price: $k^* = \tau\pi + x^* + \tau_\xi$ is the sum of $\tau\pi$, the precision of the prior beliefs, $x^*$, the precision of the private signal and $\tau\xi$, the precision of the public signal $\xi$ derived by the market price.\(^\text{10}\)

The cost of acquiring financial information affects directly the choice to be informed. The agents with an information acquisition cost greater than the endogenous threshold $\tau(\Theta)$ optimally choose to remain uninformed. Therefore, the threshold identifies the lowest ability of information acquisition according to which it is worthy to be informed and it is given implicitly by the following formula:

$$C_x(\tau, 0) = \frac{1}{2\rho R(\tau\pi + \frac{x^*}{\rho r_1(1-r^2\pi)}} \frac{1}{\rho r_1(1-r^2\pi)}$$

Let consider a policy aimed at making the illiterate agents more informed and, therefore, more active traders. The policy makers can intervene either reducing the cost of acquiring financial information of the illiterates, that can be seen as the baseline level of financial literacy, e.g. the knowledge that an individual develops in the schooling period, or increasing the fraction of literate agents, providing financial education.

In the former case, policies such as increasing the mandatory years of schooling or improving the transparency of the financial markets, affect the baseline ability of acquiring financial information. I focus on the distance between the endogenous threshold $\tau(\Theta)$ and $c_H$. It could be seen as a proxy of the access to the information market because the policy affects the behaviour of the agent only if $c_H$ moves from left to right of the threshold. Therefore, I treat the distance between the information acquisition cost of the illiterates and the triggering threshold as a measure of the per capita cost that policy makers face to make the program effective. In Table 1, I keep $c_L$ constant and I make $c_H$ to decrease, showing lower degree of illiteracy for the illiterate agents. In the first column, I report the optimal information choice of the illiterates. In the last column, positive values indicate that the information acquisition cost is higher than the threshold: greater values imply a more expensive policy to make informed the illiterates.

In the latter case, providing programs related to financial matters allows to change the fraction of the literate agents. However, the policy increases the aggregate informativeness and pushes out of the information market the

\(^{10}\)See the Appendix A for the derivation of $\xi (5.1)$ and $\tau\xi (5.4)$.
illiterates, so that with more literates around, the illiterates prefer to remain uninformed. In Table 2, I report information choices for different fraction of literates. Increasing $\lambda$ has similar effects on the aggregate informativeness $I$ and on the optimal information choice of the literates $x_L^*$ as reducing the information acquisition cost of the illiterates $c_H$. However, the effect on the optimal information choice of the illiterates is different: the illiterates become uninformed with higher $\lambda$ and informed with lower $c_H$.

This is due to the fact that acquiring information depends also on the aggregate informativeness. More information the market price reveals, less incentives the agents have to acquire information. Differentiating equation (3.1) with respect to $I$, I can show that the optimal information choice is a non increasing function of the aggregate informativeness. Formally, I have:

$$\frac{dx}{dI} = -\frac{2C_x}{C_{xx} + C_x} \frac{I\theta}{\rho(1-t)^2} \leq 0$$

which shows the strategic substitutability between private and public information.

Therefore, the policy aimed at reducing the information acquisition cost of the illiterates affects only indirectly the information choice of the literates: more aggregate informativeness induces them to reduce their acquired information: $\frac{dx}{dc_H} \geq 0$. On the other hand, the overall effect on the information choice of the illiterates is positive: lower information acquisition cost leads them to acquire more information: $\frac{dx}{dc_H} \leq 0$.$^{11}$

**The portfolio choice**

The optimal portfolio share is the standard solution for the maximization problem of an agent with CARA utility function. The agent optimally chooses to trade when she believes that the expected excess return is positive. For each agent $j \in J$, the optimal portfolio share $\alpha_j$ depends on the precision of the posterior belief, $k_j$, and on the expected excess return conditional to the agent’s informative set: $E[\pi|F_j] - pR$:

$$\alpha^*_j = \frac{p k_j}{\rho (W_1 - C_j)(1-t)} (E[\pi|s_j, p] - pR)$$

and the following condition holds: $\alpha^*_j (E[\pi|s_j, p] - pR) > 0$. To highlight the role played by the information in the portfolio choice, I rewrite the optimal portfolio share:

$$\alpha^*_j = \frac{W_j}{W_1 - C_j} \alpha^*|_{x=0} + \frac{p}{\rho(1-t)(W_1 - C_j)} [x_j(s_j - pR)]$$

where the first term is the optimal share of an uninformed agent who follows only market feelings, i.e. the public knowledge embodied in the prior beliefs $^{11}$See the appendix B.
and in the market price, herding on what the market partially reveals.\textsuperscript{12} The second term in brackets is the risky asset’s premium as predicted by the private signal. It is the extra portfolio share of the informed agent, who uses the private information to balance what the market suggests; her trading position is consistent with what her private signal suggests to do. In case she gets a private signal realization different from the market feelings, she would like to bet against the market, in order to speculate on her private knowledge. Furthermore, more precise private signal she has, more aggressively she would like to trade.

The market price variance

The equilibrium market price is a linear function of the fundamentals: \( p_R = a + b\pi - d\theta \) where the coefficients \( a, b \) and \( d \) are determined in (5.3). The equilibrium market price depends on the aggregate information \( I \) and I work out the relationship through the beliefs of the "average agent". I do not mean there exists a real agent with such beliefs. I mean a fictitious agent with private signal realization \( s_j = \pi \) and private signal precision equal to \( I \), as in a shared-information economy where the central planner can observe all the private signals taking the sample mean: \( \int s_jdj = \pi \), with precision \( \int x_jdj = I \).

The average agent has posterior precision:

\[
K = \tau_\pi + I + \tau_\xi
\]

and posterior mean:

\[
E[\pi|\pi, p] = K^{-1} [\mu_\pi \tau_\pi + \pi I + \xi \tau_\xi]
\]

After having substituted \( a, b \) and \( d \) and few algebra steps, the equilibrium market price can be rewritten as:

\[
p_R = E[\pi|s, p] - \frac{\rho(1-t)\theta}{K}
\]

It is driven by two components: the posterior belief of the average agent and the discount on the price demanded by the traders to be compensated for the uncertainty due to the noisy supply.

The market price is random because it depends on the realization of the fundamentals. Given the assumed probability distribution, the equilibrium market price is a Gaussian with mean: \( \mu_{pR} = \mu_\pi - \frac{\rho(1-t)\theta}{K}\mu_\theta \) and variance:

\[
\sigma^2_{pR} = \frac{1}{K^2} \left\{ (\tau_\xi + I)^2 \frac{1}{\tau_\pi} + \left[ \frac{I\tau_\xi}{\mu(1-t)} + \rho(1-t) \right]^2 \frac{1}{\tau_\theta} \right\}
\]

\textsuperscript{12}When \( I = 0 \), no agents purchase private information. There are only uniformed and noisy traders in the markets. The market price just reflects the noisy supply and the agents hold risky assets in order to offset it. When \( I \to \infty \), \( (1-\tau_\pi^{-1} I_{\tau_\pi}) \) goes to zero and agent \( j \) does not purchase any risky assets: \( \alpha^*_j = 0 \). The market price fully reveals the value of the fundamentals, therefore there are no reasons to trade.
Hereafter I control the impact on the variance of the market price of the changes in exogenous factors such as the uncertainty of the fundamentals and the risk aversion of the agents.

In table 3, I show how fundamentals’ uncertainty, \( \tau_\pi \) and \( \tau_\theta \), affects the market price variance. Lower uncertainty about the asset payoff and the asset supply decreases the market price variance, with wider range for the former (table 3(a)) than for the latter (table 3(b)). The market price reflects the uncertainty in the fundamentals, but the impact of the noisy in the asset supply is mitigated by the aggregate informativeness, the risk aversion and the tax rate.

Risk aversion does not monotonically affects volatility. Either with high and low risk aversion, the market price variance is higher than when the agents are medium risk averse.\(^{13}\) This result is driven by the impact of the risk aversion on the coefficients of the fundamentals in the price function. Less risk averse agents (low \( \rho \)) heavily trade and transfer their private information to the market price. The market price is more sensible to the asset payoff shocks (high \( b \)) and is less sensible to the noisy supply shocks (low \( d \)), given the ability of the traders to absorb the shocks of the noisy supply.

Increasing the risk aversion leads the agents to acquire more private information and to reduce their trade. Therefore, the aggregate informativeness increases but the sensitivity of the market price to the asset payoff shocks decreases. Moreover, the reduced trading of the agents implies that the market dries up and the sensitivity of the market price to the noisy supply shock increases but at lower rate with respect to the decrease of the sensitivity to the asset payoff shocks. The total effect is that higher risk aversion leads to lower market price variance.

This relationship holds up to a point, according to which an increase in risk aversion let the agents reduce their acquisition of private information, given the lower value attached to it. Therefore, the aggregate informativeness decreases and reinforces the decrease of the sensitivity of the market price to the asset payoff shocks. Moreover, the direct effect of the reduced trading and the indirect effect of lower aggregate informativeness increases the sensitivity of the market price to the noisy supply shocks. The effect of the risk aversion on the market price variance changes and increasing risk aversion leads to higher volatility in the market price.

I do not go deep into the explanation of the mechanism underlying the effect of the risk aversion on the volatility of the market price. I focus on the effects of exogenous variables that are under the control of the policy makers: the tax rate through fiscal policies, the cost of acquiring information, improving transparency rules of the financial markets, and the amount of literate agents, providing financial education programs.

\(^{13}\)In Table 4 I report the market price variance for increasing degrees of risk aversion under different scenarios of uncertainty in the fundamentals.
I will discuss the fiscal policy in the next section. Hereafter, I focus on the role of policies such as implementing better transparency rules for the financial markets or improving the baseline knowledge of the population (lower $c_H$).

I show the following theorem:

**Theorem 1.** *The impact of the policy on the market volatility is cross partial non-monotone, even if the policy always increases the aggregate informativeness.*

As a proxy of the market volatility, I use the market price variance. The policy affects the information choice of the illiterate, and indirectly also that of the literate, through the aggregate informativeness. I decompose the effect to work out the region of the parameter space where the impact of the policy is well determined. Formally, the policy effect on the market stability is given by:

$$\frac{d\sigma^2_R}{dc_H} = \frac{d\sigma^2_R}{dI} \frac{dI}{dc_H}$$

where $\frac{d\sigma^2_R}{dI}$ is the impact of the aggregate informativeness on the market price variance and $\frac{dI}{dc_H}$ is the impact of the policy on the aggregate informativeness.\(^{14}\)

The latter, $\frac{dI}{dc_H}$, is always non positive. The result is due to two effects: the first effect would drive up the aggregate informativeness. The policy allows the illiterate agents to acquire information at a lower cost and, therefore, they optimally acquire more information. The second effect would drive down the aggregate informativeness: higher private information acquired by the illiterates implies higher aggregate informativeness and this leads both types to reduce their acquisition of private information. The first effect dominates the second one and the policy leads to higher aggregate informativeness. Only if it is optimal to remain uninformed for the illiterates, the impact of the policy on the aggregate informativeness is null.

The former term, $\frac{d\sigma^2_R}{dI}$, is cross partial non-monotone. In order to study it, I collapse the behaviour of the two types into the fictitious average agent I described before. The impact of the aggregate informativeness on the market price variance is negative when the market price variance is greater than a threshold, that it is given by two terms. The first one is the prior variance of the asset payoff, weighted by the sensitivity of the market price to the asset payoff shocks, i.e. $\frac{1}{\tau_w}$. The second term is the inverse of the effect of the aggregate informativeness on the variance of the average agent’s belief, weighted by the sensitivity of the market price to the noisy supply shocks, i.e. $\frac{1}{\rho(1-\tau) \frac{dK}{dI}}$. Namely, I have:

$$\frac{d\sigma^2_R}{dI} < 0 \iff \sigma^2_R > \frac{b}{\tau_w} + \frac{d}{\rho(1-\tau) \frac{dK}{dI}}$$

\(^{14}\)See the appendix B, equation (5.8).
where $\frac{dK}{dI}$ is the monotone change in the posterior precision of the average agent’s belief when the aggregate informativeness increases.

Higher aggregate informativeness decreases the market price variance if the latter is sufficiently higher than the prior variance of the asset payoff $\frac{1}{\tau}$, depending on the sensitivity of the market price to the market shocks and on the changes of the posterior variance of the average agent’s belief with respect to the aggregate informativeness. If $K$ is not much responsive to the aggregate informativeness, for market volatility to decrease as aggregate informativeness increases, it must hold that the gap between the market price variance and the prior variance of the asset payoff $(\sigma^2_{pR} - \frac{b}{\tau})$, is high enough, i.e., the contribution of the noisy supply to the overall market volatility is small.\(^\text{15}\)

I summarize the impact of the policy on market volatility with the following formula:

$$\frac{d\sigma^2_{pR}}{d\omega} > 0 \iff \sigma^2_{pR} > \frac{b}{\tau} + \frac{d}{\rho(1-t)} \frac{1}{dK/dI}$$

The policy makers can manipulate the information acquisition cost of the illiterates and this intervention affects the aggregate informativeness through the agents’ behaviour. The behaviour of the fictitious average agent helps at illustrating the effect of the policy on market stability, which depends on the gap between the market price variance and prior variance of the asset payoff, and on the sensitivity of the posterior belief variance with respect to the aggregate informativeness.

The main intuition is that implementing a policy aimed at lowering the information costs for the illiterate agents leads them to acquire more private information and to trade more actively. Therefore, more private information revealed by the market price affects positively market volatility. On the other hand, with low uncertainty in the fundamental, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively.

In table 5 I report the market price variance for increasing values of the financial information acquisition costs of the illiterates and in figure 2, I plot the variance of the market price and the threshold for different values of the fundamentals uncertainty: $\tau_\phi, \tau_\theta = [0.3, 0.5, 1.2]$. When the market price variance is above the threshold, then it increases in the inequality of the financial information acquisition costs between the two types of agents. On the other hand, when the variance is lower than the threshold, it decreases in the inequality.

Also a policy aimed at increasing the fraction of the literate agents ($\lambda$) obtains the same effect on the variance of the market price. Providing financial education to the illiterates makes them literate and the effect of having\(^\text{15}\)Recall that the market volatility reflects both the volatility of the asset and of the noisy supply, given the assumed market structure.
an increased amount of literate traders is cross partial non-monotone. As it is shown in table 6, the market price variance decreases in λ when there is high fundamentals’ uncertainty and increases with low uncertainty.

The expected excess return

The incentive to trade is the expected excess return: \( f = E[\pi|\mathcal{F}] - pR \). At the planning period, it is random variable made by the equilibrium market price and by the posterior beliefs about the payoff. The latter is itself a random variable normally distributed with mean and variance:

\[
E[E[\pi|\mathcal{F}]] = \mu_\pi \quad \text{Var}[E[\pi|\mathcal{F}]] = \frac{1}{\tau_n} - \frac{1}{k}
\]

The expectation is the prior mean of the risky asset and it is the same for both types. The variance of the posterior beliefs differs between types: it is higher for literates than for illiterates. Literates purchase more information and they end up, on average, with posterior beliefs close to the true value. The illiterate agents rely less on their private signals and their posterior beliefs are, on average, closer to the market price.

Thus, I compute the distribution of the expected excess return: \( f \sim N(\mu_f, \sigma_f^2) \):

\[
\mu_f = \rho \mu_\theta K^{-1} \quad \sigma_f^2 = \frac{d^2}{\tau_\theta} + \frac{1}{K} \left( \frac{\tau_\pi}{K} - \frac{K}{k} \right)
\]

The expected gain from being a trader is given by the mean of the noisy supply scaled by the risk aversion and the posterior beliefs precision of the average agent. It comes from the opportunity to use the informational advantages against those agents who need to trade for exogenous reasons. It is decreasing in the aggregate informativeness.\(^{16}\)

The variance of the expected excess return differs between types. There is a common part \( \frac{d^2}{\tau_\theta} \) that refers to the volatility of the noisy supply scaled by the square of the sensitivity of the market price to the shocks of the noisy supply, i.e., the market depth. Lower volatility of the noisy supply and higher liquidity of the market imply lower volatility of the expected excess return.

The second part, \( \frac{1}{K} \left( \frac{\tau_\pi}{K} - \frac{K}{k} \right) \), shows the informational gain achieved by one type with respect to the average agent. Those with more precise posterior beliefs take into account greater return due to the opportunity to use their informational advantages. However, the informational advantages decrease as the aggregate informativeness increases.

\(^{16}\)With fully revealing price \( (I = \infty) \), the expected gain is zero. With less revealing price \( (0 < I < \infty) \), the opportunity to take advantage of the noisy traders is shared with less traders, increasing the per capita expected gain.
4 Welfare analysis

In this section, I use the model to derive the policy implications of financial education programs. Let the policy makers provide financial education to a given amount of agents \( \lambda \), making them more financial literate and able to acquire financial information at a lower cost.

I assume that the expenditures of the financial education program is equal to \( \lambda \) plus a factor that it is proportional to the distance between the current ability and the earned ability after having attended the program \( (c_H - c_L) \). The policy expenditures is financed through general taxation: the policy makers set the tax rate \( t \) on the excess return of the traders. The budget constraint is satisfied in expectation terms over the joint distribution of the fundamentals: \(^{17}\)

\[
E[t \int_j \max\{CG_j, 0\}dj] = \lambda + (c_H - c_L)
\]

with \( CG = \alpha_j^\ast (W_1 - C_j^\ast) \frac{\pi-p_R}{p} \). In the model, \( \lambda \) does not only capture the cost of the policy, but also affects the incentive to become informed and to trade actively. On the other hand, the ability acquired by the agents who attend the financial education program, \( c_L \), affects their information choice, but also the information choice of the illiterates, through the aggregate informativeness. I study the impact of the policy moving along these two dimensions: the fraction of the literates and the inequality in financial information acquisition cost. In Table 7 and Table 8 I report the tax rate that satisfies the public budget constraint.

In Table 7, I keep \( \lambda = 0.25 \) and I let the inequality in the financial information acquisition costs increase, i.e. higher productivity of the financial education program due to higher quality of the courses. The results show an increasing tax rate in the inequality. Increasing the ability of the agents who attend the financial education program implies higher aggregate informativeness due to their information choice. The feed-back effect is to decrease the expected fiscal revenue, so that the policy makers need to set a higher tax rate to finance the policy expenditures. The expected utility of the literates increases due to the informational benefit, while that for the illiterate decreases, driving down the aggregate welfare. In figure 3 I report the expected utility of the two types and a measure of the aggregate welfare \( WU \) given by the weighted sum of the individuals' expected utility. The left panel shows lower aggregate utility for increasing inequality in the information acquisition costs.

In Table 8, I keep \( c_L = 0.01 \) and I let the fraction of the literates increase. The results show higher tax rate with more literates. The impact of the policy on the aggregate informativeness is non monotone and depends on

\(^{17}\)See the appendix C for the computations and the numerical results.
the changes on $\lambda$ and on the information choices of the two types. Higher $\lambda$ implies a higher contribute of the literates to the aggregate informativeness and, at the same time, lower information acquired, either due to the strategic substitutability and to the lower return of the information acquisition. The right panel of figure 3 shows decreasing expected utility of both types for increasing fraction of the literates.

Thus, a policy aimed at improving financial literacy providing financial education program is not welfare improving if the policy makers introduce a proportional tax on the excess return to finance the policy expenditures.

5 Conclusion

This paper adopts a noisy rational expectations equilibrium with endogenous information acquisition to analyse heterogeneity in financial literacy and its impact on market volatility. The model provides rationale for the existence of financial education programs, aimed at improving individual financial literacy. Moreover, the welfare analysis identifies a tax rate that allows to finance the financial education programs and checks the effects on the individual and aggregate welfare.

In the model, I posit that financial literacy affects individual information acquisition costs. The individuals choose how much information to acquire and the share of wealth to invest in risky assets. There are two types of individuals, literate and illiterate, and they trade against noisy traders. I point out that the more financially literate individuals are, the more information they acquire and the more information is revealed by the market price. Within a general equilibrium framework, I derive the market volatility and the implications of policies aimed at improving the financial literacy of the agents.

For a policy that improves the transparency of the financial markets or increases the baseline knowledge of the population, improving the ability to acquire and process information, the policy makers should take into account a cross-partial non-monotone effect on market volatility. The effect depends on uncertainty in the market fundamentals. The policy implication is to reduce the information acquisition cost only when there is high uncertainty in the market fundamentals. The main intuition is that lower information cost for the less literate households leads them to acquire more private information and to trade more actively. Therefore, more private information revealed by the market price affects positively the market volatility. On the other hand, with low uncertainty in the fundamental, the positive informational effect is offset by the negative effect of having more traders with less precise beliefs, who trade more conservatively.

If the policy makers provide financial education programs financed through general taxation on the market excess returns, the model shows a decreasing
aggregate welfare in the population coverage and in the productivity of the program, due to the higher tax rate needed to finance the policy expenditures. Only in the latter case, the expected utility of the agents who attend the program increases because the informational benefit of the programs are greater than the taxes to be paid.

Further research will take into account participation costs in order to explain limited market participation and let the individual amount of financial literacy be endogenously chosen by the agents.
Appendix A - Proof of proposition 1

In order to prove the existence and the uniqueness of the equilibrium, I need to prove the existence and the uniqueness of an equilibrium market price that linearly depends on the fundamentals, \( \pi \) and \( \theta \), and clears the market. In equilibrium, all the agents maximize their utility. I compute optimal asset demands and the equilibrium market price for given aggregate informativeness \( I \). In the following step of the proof, I derive the optimal information choices and I work out the equilibrium in the information market, proving the existence and the uniqueness of the solution of the model.

Lemma 1 (Individual asset demands and Market Price). For given aggregate informativeness \( I \), the optimal portfolio share for agent \( j \in J \) is given by:

\[
\alpha^*_j = \frac{k_j p}{\rho[W_1 - C(x_j, \epsilon_j)](1 - t)} (E[\pi|F_j] - pR)
\]

and the equilibrium market price is \( pR = a + b\pi - d\theta \) where the coefficients \( a, b \) and \( d \) are determined in (5.3).

Proof. The proof is given in five steps. In the first step, I guess a price linear function and I derive the informationally equivalent public signal \( \xi \) from the price function. In the second step, I compute the mean and the variance of the posterior beliefs given the unbiased signals, \( \xi \) and \( s \). In the third step, I derive the optimal asset demand. In the fourth step, I derive market clearing conditions and, in the last step, I impose rationality and determine the coefficients of the guessed linear price function.

Recall that the joint distribution of the payoff, the supply and the signals is:

\[
\begin{pmatrix}
\theta \\
\pi \\
s_j \\
\end{pmatrix} 
\sim \mathcal{N}
\begin{pmatrix}
\begin{pmatrix}
\mu_\theta \\
\mu_\pi \\
\mu_s \\

\end{pmatrix} \\
\begin{pmatrix}
\frac{1}{\tau_\theta} & 0 & 0 & \ldots & 0 \\
0 & \frac{1}{\tau_\pi} & -\frac{1}{\tau_{\pi\pi}} & \ldots & 0 \\
0 & -\frac{1}{\tau_{\pi\pi}} & \frac{1}{\tau_{\pi\pi\pi}} & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & 0 \\
0 & \ldots & \ldots & \ldots & \frac{1}{\tau_{\pi\pi\pi\pi}} \\
0 & \ldots & \ldots & \ldots & \frac{1}{\tau_{\pi\pi\pi\pi\pi}} \\

\end{pmatrix}
\end{pmatrix}
\]

**Step 1** : The agents guess a price function linear in \( \pi \) (future payoff) and \( \theta \) (noisy supply):

\[
pR = a + b \left( \lambda \int_{j \in L} s_j dj + (1 - \lambda) \int_{j \in H} s_j dj \right) - d\theta
\]

Applying the law of large number within each group of traders, I have that \( \int s_j \epsilon_j dj = 0 \) with probability one. Therefore, I can rewrite the price function as:

\[
pR = a + b\pi - d\theta
\]
The agents use the observed private signals to update their prior beliefs \( \pi \sim N(\mu_\pi, \tau_\pi^{-1}) \). The private signal is unbiased by construction, \( s | \pi \sim N(\pi, x^{-1}) \), and conditionally independent from the prior belief, \( \mu_\pi, E[(\mu_\pi - \pi)(s - \pi)] = 0 \). Rational agents use the price as a public signal. It is not unbiased:

\[
E[pR|\pi] = a - d\mu_\theta + b\pi.
\]

To apply Bayesian updating, the agents transform the price in an informationally equivalent variable \( \xi \):

\[
\xi = \frac{pR - a + d\mu_\theta}{b} = \pi - \frac{d}{b}(\theta - \mu_\theta)
\]

that is unbiased, \( \xi | \pi \sim N(\pi, \tau_\theta^{-1}) \).

**Step 2**: Each agent \( j \) observes \( F_j = \{s_j, p\} \equiv \{s_j, \xi\} \) and updates her prior beliefs with the two Gaussian signals. Using the formula for the multivariate normal distribution (Degroot (2004), p. 55), the posterior expectation is given by:

\[
E[\pi|s_j, p] = \mu_\pi + 1 \left\{ x_j (s_j - E[s_j]) + \frac{b^2}{\sigma_\theta^2} \tau_\theta (\xi - E[\xi]) \right\}
\]

where the precision of the posterior belief \( k_j \) is given by the sum of precisions of the prior, of the private signal and of the public signal:

\[
k_j = \frac{1}{\text{Var}[\pi|s_j, p]} = \tau_\pi + x_j + \frac{b^2}{\sigma_\theta^2} \tau_\theta
\]

**Step 3**: Maximizing the CARA utility function with respect to the control variable \( \alpha \), each agent solves the following:

\[
\max_\alpha E \left[-\frac{1}{\rho} e^{-\rho W_2(\alpha)}|s, p\right] = \max_\alpha -\frac{1}{\rho} e^{-\rho E[W_2(\alpha)|s, p]} - \frac{D}{2} \text{Var}[W_2(\alpha)|s, p]\] (5.2)

The final wealth can be rewritten as:

\[
W_2 = \begin{cases} 
\alpha \frac{\pi - pR}{p} + R \right) (W_1 - C) & \text{if } \alpha (\pi - pR) < 0 \\
\alpha \frac{\pi - pR}{p}(1 - t) + R \right) (W_1 - C) & \text{if } \alpha (\pi - pR) > 0
\end{cases}
\]

where the first condition identifies the losses and the second one the gains. Only the capital gains are affected by the tax rate.

---

18I use the log-normal distribution properties and I drop subscript \( j \) to simplify notation. I will restore it when I aggregate the individual asset demands.
Before $\pi$ is revealed and after having observed the private signal, the final wealth is a random variable and the conditional expectation is:

$$E[W_2|s,p] = \begin{cases} \left[ \frac{\alpha}{\lambda} \frac{E[\pi|s,p] - \rho R}{p} + R \right] (W_1 - C) & \text{if } \alpha (E[\pi|s,p] - \rho R) < 0 \\ \left[ \frac{\alpha}{\lambda} \frac{E[\pi|s,p] - \rho R}{p} (1 - t) + R \right] (W_1 - C) & \text{if } \alpha (E[\pi|s,p] - \rho R) > 0 \end{cases}$$

while the variance is:

$$Var[W_2|s,p] = \begin{cases} \frac{\alpha^2}{\lambda^2} (W_1 - C)^2 Var[\pi|s,p] & \text{if } \alpha (E[\pi|s,p] - \rho R) < 0 \\ \frac{\alpha^2}{\lambda^2} (1 - t)^2 (W_1 - C)^2 Var[\pi|s,p] & \text{if } \alpha (E[\pi|s,p] - \rho R) > 0 \end{cases}$$

After having substituted the expectation and the variance of the final wealth in (5.2), we can solve the maximization problem. The optimal risky asset demand for agent $j$ is:

$$\alpha_j^* = \frac{p k_j}{\rho (W_1 - C)} \frac{E[\pi|s_j,p] - \rho R}{(E[\pi|s_j,p] - \rho R)}$$

and the following condition holds: $\alpha_j^* (E[\pi|s_j,p] - \rho R) > 0$.

The amount of wealth in risky assets depends on the posterior precision $k_j$, the risk aversion $\rho$, the tax rate $t$ and the expected excess return of the risky investment.

**Step 4**: The equilibrium price clears the market for the risky asset. Aggregating over all traders yields the aggregate demand:

$$\int_{j \in L} \alpha_j^* \frac{W_1 - C_j}{p} dj + \int_{j \in H} \alpha_j^* \frac{W_1 - C_j}{p} dj = \theta$$

I apply the weak law of large numbers for independent and identically distributed random variables with the same mean, such that $\int_{j \in L,dj} = \pi$. Then, imposing market clearing condition holds the following equation:

$$\left[ \lambda \left( \mu_\pi \tau_\pi + x_L \pi + \frac{b^2}{\pi^2} \tau_0 \xi - \rho R k_L \right) + (1 - \lambda) \left( \mu_\pi \tau_\pi + x_H \pi + \frac{b^2}{\pi^2} \tau_0 \xi - \rho R k_H \right) \right] = \rho (1 - t) \theta$$

$$\mu_\pi \tau_\pi + \frac{b^2}{\pi^2} \tau_0 \xi + \pi [\lambda x_L + (1 - \lambda) x_H] - \rho R [\lambda k_L + (1 - \lambda) k_H] = \rho (1 - t) \theta$$

where $x_L$ and $k_L$ are the information choice and posterior precision of the literate agent (type $L$). Similarly, $x_H$ and $k_H$ for the illiterate agent (type $H$). Using the definition of aggregate informativeness $I = \lambda x_L + (1 - \lambda) x_H$, I can rewrite the price equation as

$$pR = \left( \tau_\pi + I + \frac{b^2}{\pi^2} \tau_0 \right)^{-1} \left[ \mu_\pi \tau_\pi + \frac{b^2}{\pi^2} \tau_0 \xi + \pi I - \rho (1 - t) \theta \right]$$
Step 5: I impose rationality. ξ involves undetermined coefficients b, d. I substitute the expression for ξ = π - \frac{d(\theta - \mu)}{b} and, rearranging the terms, I have:

\[ pR = (\tau_\pi + I + \frac{b^2 \tau_\theta}{\rho^2(1-t)^2})^{-1} \left\{ \mu_\pi \tau_\pi + \frac{b}{\rho^2(1-t)^2} \mu_\theta + \pi(I + \frac{b^2 \tau_\theta}{\rho^2(1-t)^2} - \theta \left[ \frac{b}{\rho^2(1-t)} + \rho(1-t) \right]) \right\} \]

I derive \( \frac{b}{\rho} = \frac{I}{\rho^2(1-t)^2} \) and I substitute it back into the price function. I find out the following determined coefficients.

\[
\begin{align*}
    a &= \frac{\mu_\pi \tau_\pi + \frac{I}{\rho^2(1-t)^2} \mu_\theta \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta} \\
    b &= \frac{I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta} \\
    d &= \frac{\rho + \frac{I}{\rho^2(1-t)^2} \tau_\theta}{\tau_\pi + I + \frac{I^2}{\rho^2(1-t)^2} \tau_\theta}
\end{align*}
\]  

(5.3)

To simplify the notation, I rewrite the precision of the public signal ξ:

\[
\tau_\xi = \frac{I^2}{\rho^2(1-t)^2} \tau_\theta
\]

(5.4)

Lemma 2. For given aggregate informativeness I, for all the agents with \( c < c(\Theta) \), the optimal information choice \( x^* \) solves the following equation:

\[
2\rho RC_x(x^*, c)(\tau_\pi + x^* + \tau_\xi) = 1
\]

(5.5)

where \( c(\Theta) \) is given by (5.7).

Proof. In order to solve for the information choice \( x^* \), first I need to compute the indirect utility: \( v(s_j, p; \Theta) = E[U(W_2(\alpha_j^*))|s_j, \xi] \). Then, I need to compute the expected value. It depends on the first two moments of the expected return of trading a unit of risky asset. Once I derive the expected indirect utility function, I apply the concave maximum theorem to find out the optimal information choice \( x^* \) and to identify the threshold \( c(\Theta) \), according to which is worthy to be informed.

Step 1 The outcome of the first step is the indirect utility function. For agent \( j \in J \), it is:

\[
v(s_j, p; \Theta) = -\frac{1}{\rho} e^{-\frac{1}{2}k_j(E[\xi | s_j, p] - pR)^2} - \rho R[W_1 - C(x_j, c_j)]
\]

I derive it substituting the optimal asset demand back into the expected utility (5.2). Thus, for each agent:

\[
v(s, p; \Theta) = E[-\frac{1}{\rho} e^{-\rho W_2(\alpha^*)} | \mathcal{F}] - \frac{1}{\rho} e^{-\rho(E[W_2(\alpha^*) | s, \xi]) - \frac{\theta^2}{2} Var[W_2(\alpha^*) | s, \xi]}
\]

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where

$$E[W_2(\alpha^*)|s, \xi] = E\left[ \alpha^* \frac{p - pR}{p} (1 - t) + R \left[ W_1 - C(x, c) \right] | s, \xi \right]$$

$$= \left[ W_1 - C(x, c) \right] E[\left( \alpha^* \frac{p - pR}{p} (1 - t) + R \right) | s, \xi]$$

$$= \left[ W_1 - C(x, c) \right] \left( \frac{p}{p[W_1 - C(x, c)](1 - t)} E[\pi|s, p] - pR \right) \frac{E[\pi|s, p] - pR}{p} (1 - t) + R$$

$$= \frac{k}{p} (E[\pi|s, p] - pR)^2 + R[W_1 - C(x, c)]$$

and:

$$Var[W_2(\alpha^*)|s, \xi] = \frac{[W_1 - C(x, c)]^2(1-t)^2(\alpha^*)^2}{p^2} Var[\pi|s, p] = \frac{[W_1 - C(x, c)]^2 p^2 k^2 \left( E[\pi|s, p] - pR \right)^2}{p^2 \left( W_1 - C(x, c) \right)^2 (1-t)^4} \frac{1}{k}$$

Substituting back the last two terms into the indirect utility of agent \( j \), I get:

$$v(s_j, p; \Theta) = -\frac{1}{p} e^{-\rho \left( \frac{k}{p} (E[\pi|s, \xi] - pR)^2 + R[W_1 - C(x, c, j)] \right)} - \frac{1}{2} k_j \left( E[\pi|s, \xi] - pR \right)^2 - R[W_1 - C(x, c, j)]$$

$$= -\frac{1}{p} e^{-\frac{1}{2} k_j (E[\pi|s, \xi] - pR)^2 - pR[W_1 - C(x, c, j)]}$$

Once I derive the indirect utility, I need to compute the expected value. For simplifying notation, I call \( f \) the expected return: \( f = E[\pi|s, p] - pR \) that is normally distributed with mean \( \mu_f \) and variance \( \sigma_f^2 \).

**Step 2** The outcome of the second step is the expected value of the indirect utility function. For agent \( j \in J \), it is:

$$E[v(s_j, p; \Theta)] = -\left( \frac{1}{k_j} + \sigma_f^2 \right) \left( \frac{1}{k_j} + \sigma_f^2 \right)^{-1/2} e^{-\frac{1}{2} \sigma_f^2 - \rho R[W_1 - C(x, c, j)]}$$

The expected return \( f = E[\pi|s, p] - pR \) is a normal random variable, given that is a linear function of two normally distributed random variables. In order to compute the expected value of the indirect utility function, I need to compute the mean, \( \mu_f = E[E[\pi|s, p] - pR] \), and the variance, \( \sigma_f^2 = Var[E[\pi|s, p] - pR] \), of the expected excess return. I start computing the expectation of the posterior belief:

$$E[E[\pi|s, p]] = E\left[ \mu_\pi + \frac{1}{k} [x (s - E[s]) + \frac{1}{\sqrt{\gamma}} (\xi - E[\xi])] \right]$$

and the expectation of the market price:

$$E[pR] = E[a + b\pi - d\theta]$$

$$= a + b\mu_\pi - d\mu_0$$

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Therefore, the mean of the expected excess return is:

\[
\mu_f = \mu_x - a - b\mu_x + d\mu_\theta
\]

\[
= (1 - b)\mu_x - a + d\mu_\theta = \frac{b\mu_\theta}{K} = \mu_f
\]

where \(K = \tau_x + I + \tau_\xi\).

To compute the variance, I need the variance of the posterior belief, the market price variance and the covariance of two terms:

\[
\sigma_f^2 = Var[E[\pi|s,p]] + Var[pR] - 2Cov[E[\pi|s,p], pR]
\]

where the variance of the posterior belief is:

\[
Var[E[\pi|s,p]] = \frac{x^2}{k^2}(\frac{1}{\tau_x} + \frac{1}{x}) + \frac{t^4\tau_x}{\rho^2(1-t)^2} \left[ \frac{1}{\tau_x} + \frac{\rho^2(1-t)^2}{l^2\tau_\theta} \right] + 2\frac{x^2 l^2\tau_\theta}{\rho^2(1-t)^2} k^2 \left( \frac{1}{\tau_x} \right)
\]

\[
= \frac{1}{\tau_x} k^2 \left[ x^2 + x\tau_x + \frac{t^4\tau_x}{\rho^2(1-t)^2} + \frac{\rho^2(1-t)^2}{l^2\tau_\theta} + 2x^2 l^2\tau_\theta \right]
\]

\[
= \frac{1}{\tau_x} k^2 \left[ x + \frac{t^2\tau_\theta}{\rho^2(1-t)^2} \right] k
\]

\[
= \frac{1}{\tau_x} - \frac{1}{k}
\]

The market price variance is:

\[
Var[pR] = b^2 \frac{1}{\tau_x} + d^2 \frac{1}{\tau_\theta}
\]

and the covariance between posterior beliefs and market price is:

\[
Cov[E[\pi|s,p], pR] = b \frac{1}{\tau_x}
\]

Thus, I can rewrite \(\sigma_f^2\) as:

\[
\sigma_f^2 = \frac{1}{\tau_x} - \frac{1}{k} + b^2 \frac{1}{\tau_x} + d^2 \frac{1}{\tau_\theta} - 2b \frac{1}{\tau_x}
\]

\[
= (1 - b)^2 \frac{1}{\tau_x} + d^2 \frac{1}{\tau_\theta} - \frac{1}{k}
\]

\[
= \frac{d^2}{\tau_\theta} + \frac{1}{K} \left( \frac{\tau_x}{K} - \frac{K}{\tau_x} \right)
\]

Once I have the first two moments of the excess return, I can compute the expected value of the indirect utility:

\[
E[v(s,p; \Theta)] = E[-\frac{1}{\rho} e^{-\frac{1}{2} k f^2 - \rho R[W_1 - C(x,c)]}]
\]

\[
= -\frac{1}{\rho} e^{-\rho R[W_1 - C(x,c)]} E[e^{-\frac{1}{2} k \sigma_f^2 (\frac{f}{\sigma_f})^2}]
\]

Recall that \(f \sim N(\mu_f, \sigma_f^2)\) and \((\frac{f}{\sigma_f})^2 \sim \chi_1^2\). Therefore, I can use the moment generating function of a non central \(\chi_1^2\). This is given by the following formula:

\[
M(t, h, \lambda) = E[e^{i\zeta}] = e^{\frac{\lambda^2}{2t}} \frac{e^{\frac{\lambda^2 h}{(1 - 2t)^{3/2}}}}{(1 - 2t)^{h/2}}
\]
In my case, I have:

\[ h = 1 \]
\[ t = -\frac{1}{2} k \sigma_f^2 \]
\[ \lambda = \left( \frac{\mu_f}{\sigma_f} \right)^2 \]
\[ 1 - 2t = k(\frac{1}{k} + \sigma_f^2) \]

Therefore,

\[
E \left[ e^{-\frac{1}{2} k \sigma_f^2 (\sigma_f^2)} \right] = \left[ k(\frac{1}{k} + \sigma_f^2) \right]^{-1/2} \frac{1}{e^{\frac{1}{2} k \sigma_f^2 (\sigma_f^2)}}
\]
\[
= \left[ k(\frac{1}{k} + \sigma_f^2) \right]^{-1/2} \frac{1}{e^{\frac{1}{2} \sigma_f^2}}
\]

Where the exponential term does not depend on \( x \), given that \( (\frac{1}{k} + \sigma_f^2) \) is constant and equal to \((1 - b)^2 \frac{1}{\tau_w} + d^2 \frac{1}{\tau_w} \). Now, rearranging all the terms, the expected value of the indirect utility for agent \( j \) is:

\[
E[v(s_j, p, \Theta)] = -\frac{1}{\rho} \left[ k_j \left( \frac{1}{k_j} + \sigma_f^2 \right) \right]^{-1/2} \frac{1}{e^{\frac{1}{2} k_j (\sigma_f^2)}} - \rho R_W - C(x_j, c_j)
\]

**Step 3** The outcome of the third step is the optimal information choice \( x^* \) and the threshold \( c(\Theta) \).

I called \( \gamma \) the positive expression that is independent from the control variable \( x \):

\[
\gamma = \frac{1}{\rho} \left( \frac{1}{k} + \sigma_f^2 \right)^{-1/2} \frac{1}{e^{\frac{1}{2} \frac{\mu_f^2}{\sigma_f^2}}} - \rho R_W
\]

and I rewrite the expected value of the indirect utility function as:

\[
E[-\frac{1}{\rho} e^{-\rho W_2}] = -\gamma k^{-1/2} e^{\rho R_C(x, c)} = -\gamma (\tau_x + x + \tau_x)^{-1/2} e^{\rho R_C(x, c)}
\]

The objective function (5.6) is strictly concave and defined over a compact domain \([0, \pi(c)]\) where \( \pi(c) \) solves \( W_1 = C(\pi, c) \). The concave maximum theorem guarantees the existence and the uniqueness of the solution. It could be an interior or a corner solution: \( x^* = 0 \) or \( x^* = \pi(c) \).

I specify conditions over parameter space in order to characterize the solution. First, I derive FOC and I compute it at \( x = 0 \):

\[
\frac{E[v]}{\partial x} \bigg|_{x=0} = -\gamma (\tau_x + \tau_x)^{-1/2} \left[ \rho R_C(x, c) - \frac{1}{2(\tau_x + \tau_x)} \right]
\]

When it is positive, agent has incentive to acquire information, \( x^* > 0 \). I call \( \pi(\Theta) \) the value of \( c \), such that the agent is indifferent between being informed or remain uninformed. Formally, \( \pi(\Theta) \) is implicitly given by:

\[
[2\rho R(\tau_x + \tau_x)]^{-1} = C(x; \pi)
\]
Therefore, given strictly convexity of the cost function, \( \forall c < \bar{c}, x^*(I; c, \Theta) > 0 \).

For an interior solution, it is enough to show that there exists an \( x \in [0, \pi(c)] \) such that FOC is negative. Formally, I check when the following condition holds:

\[
\frac{1}{2\rho R(x^* + \tau_\pi + \tau_\xi)} < C_x(\pi, c)
\]

I identify a second threshold \( \underline{c}(\Theta) \) that it is implicitly given by:

\[
\frac{1}{2\rho R(x(\underline{c}) + \tau_\pi)} = C_x(\pi(\underline{c}), \underline{c})
\]

For all \( \underline{c} < c < \bar{c}, x^*(I; c, \Theta) \) is an interior solution belonging to the set \([0, \pi] \) and it is given by:

\[
2\rho RC_x(x^*, c)(\tau_\pi + x^* + \tau_\xi) = 1
\]

I derive implicitly the amount of information \( x^*(I; c, \Theta) \) that an agent optimally acquires. It depends on the financial literacy cost \( c \) and on the aggregate informativeness \( I \).

The agent is indifferent between being informed or uniformed when \( I \) goes to infinity (fully revealing market price) or if \( c = \underline{c} \). For any \( c > \underline{c} \), the optimal information choice is:

\[
x^*(I; c, \Theta) = \begin{cases} 0 & \text{if } c > \bar{c} \\ \hat{x} & \text{if } c < \bar{c} \end{cases}
\]

\[
\hat{x} = \frac{1}{2\rho R(x^*(c) + \tau_\pi + \tau_\xi)} = \frac{1}{2\rho R(x^*(c) + \tau_\pi)}
\]

In order to prove the existence and the uniqueness of a noisy rational expectation equilibrium, I need to check that the aggregation of the optimal information choices of the agents is equal to the amount of aggregate informativeness that agents conjecture in their maximization problems.

**Lemma 3.** The optimal information choices of the agents are consistent with the aggregate informativeness.

**Proof.** Let agents be distributed over \( J \) according a cdf \( G(j) \). Let \( c_j \) be the financial literacy of agent \( j \in J \). Let assume that \( c_j \) can take any values greater than \( \underline{c}(\Theta) \).

Let the compact set \( Y = [0, \frac{1}{2\rho R(c_j(0, \underline{c}))}] \subset \mathbb{R}^+ \) the domain of the following mapping operator \( F : Y \to \mathbb{R}^+ \):

\[
F(y) = \int_j x^*(c_j, y; \Theta) \text{d}j
\]

---

19. I want to avoid the case where agents prefer to spend their whole initial wealth in the information market and nothing in the asset market. This case is possible given the form of the CARA utility function where agents take into account both the mean and the variance of the final wealth.

20. In the paper, I assume that \( G(j) \) is a discrete distribution with mass \( \lambda \) on \( c = c_L \) and mass \( (1 - \lambda) \) on \( c = c_H \).
where $y$ is an element of $Y$ and $x^*(c_j, y; \Theta)$ is given by:

$$
x^*(c_j, I, \Theta) = \begin{cases} 
0 & \text{if } c_j > \tau \\
\hat{x} + \frac{2RC_x(\hat{x}, c_j)}{2\rho C_x(x^*, \rho)} - \frac{1}{2\rho C_x(x^*, \rho)} & \text{if } 0 \leq c_j \leq \tau \\
\frac{1}{2\rho C_x(0, \rho)} & \text{if } c_j < 0
\end{cases}
$$

Continuity of $F(y)$ is guaranteed by the assumption that $C_x$ is continuous. I need to prove that $F$ maps into itself to apply fixed-point Brouwer’s theorem ($F(y) = y$). For all $c_j > \zeta$ and $y \in Y$, $x^*(c_j, y; \Theta) \geq 0$. Moreover, given strictly convexity of the cost function, $C_x(0, \zeta) \leq C_x(x^*, c_j)$. This implies that:

$$
x^* = \frac{1}{2\rho C_x(0, \rho)} - \frac{1}{2\rho C_x(x^*, \rho)} \leq \frac{1}{2\rho C_x(0, \rho)}
$$

For all $c_j > \zeta$ and $y \in Y$,

$$0 \leq x^*(c_j, y; \Theta) = \max\{0, x^*\} \leq \frac{1}{2\rho C_x(0, \rho)}$$

Aggregating over $j$ using the cdf $G(j)$ implies that:

$$0 \leq F(y) = \int x^*(c_j, y; \Theta) dj \leq \frac{1}{2\rho C_x(0, \rho)}$$

Thus, $F(y)$ maps into itself and I proved the existence of the equilibrium.

To prove uniqueness of the equilibrium I follow Peress (2004). I can rewrite $N = \{j : c_j \in [\zeta, \tau]\}$ and I know that $x^*(c_j, y; \Theta) > 0, \forall j \in N$. Therefore, $\int_{j \in N} x^*(c_j, y; \Theta) dj = \int_{\zeta}^{\tau} x^*(c_j, y; \Theta) dj$.

Let $f(y) = y - \int_{\zeta}^{\tau} x^*(c_j, y; \Theta) dj$. The equilibrium value $y^*$ is a root of $f(y)$ and, to be uniquely determined, I need monotonicity of $f(y)$.

Total differentiation of $f(y)$ yields:

$$f'(y) = 1 - \left( \int_{\zeta}^{\tau} \frac{\partial x^*(c_j, y; \Theta)}{\partial y} dj + x^*(\zeta, y; \Theta) \frac{\partial C_x}{\partial y} - x^*(\tau, y; \Theta) \frac{\partial C_x}{\partial y} \right)$$

The first term in brackets is the integral of the partial derivative of the optimal information choice with respect to the aggregate informativeness. The second term in brackets is zero given the optimal choice $x^*$ is zero for agents with $c_j = \tau$. The last term is also zero given that I set $\zeta$ independent with respect to $y$.

Differentiating FOC I have:

$$C''_{xx} \frac{\partial x^*_j}{\partial y} \left( x^*_j + \tau_\pi + \frac{y^2}{\rho^2(1-\rho)^2} \tau_\theta \right) + C'_x \left( \frac{\partial x^*_j}{\partial y} + 2 \frac{y}{\rho^2(1-\rho)^2} \tau_\theta \right) = 0$$

$$\frac{\partial x^*_j}{\partial y} \left[ C''_{xx} \left( x^*_j + \tau_\pi + \frac{y^2}{\rho^2(1-\rho)^2} \tau_\theta \right) + C'_x \right] + 2C'_x \frac{y}{\rho^2(1-\rho)^2} \tau_\theta = 0$$

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\[
\frac{\partial x_i}{\partial y} = - \frac{2C'_x \frac{u}{\mu(1-u)^2} \tau \theta}{C''_{xx} \left(x_j + \tau \pi + \frac{y^2}{\mu(1-u)^2} \tau \theta \right) + C'_x} \leq 0
\]

As long as the assumptions about the shape of the cost function hold and given \( y \in Y \subset \mathbb{R}^+ \), I can conclude that \( f'(y) \) is always non-negative and \( f(y) \) is monotone. Therefore, there exists a unique value of \( y \) such that the information market is in equilibrium. \( \Box \)
Appendix B - Tables and Graphs

If it is not differently specified in the text, the model's parameters are the following: prior mean $\mu_\pi$ and prior precision $\tau_\pi$ are both equal to one. The noisy asset supply has mean zero and variance one. The risk free asset is zero return ($R = 1$) and the risk aversion coefficient ($\rho$) is one. Initial wealth ($W_1$) is one. Literate agents are a fourth of the total ($\lambda = 0.25$) and their cost of acquiring information is $c_L = 0.01$, while the information acquisition cost for the illiterates is $c_H = 0.03$. The tax rate is set to zero. In short notation, I have that $\Theta$ is given by $\{ R = 1, W_1 = 1, \rho = 1, \mu_\pi = 1, \mu_\theta = 0, \tau_\pi = 1, \tau_\theta = 1, \lambda = 0.25, c_L = 0.01, c_H = 0.03, t = 0 \}$.

Information choice

Reducing the information acquisition cost of the illiterates increases the acquired information of the illiterates and decreases that of the literates. Deriving (3.1) with respect to $c_H$ holds:

$$\left( C_{xx} \frac{dx}{dc_H} + C_{xc} \right) k_H + C_x \left( \frac{dx}{dc_H} + \frac{2\tau_\theta}{\rho^2(1-t)^2} \frac{dI}{dc_H} \right) = 0$$

Then, substituting $\frac{dI}{dc_H} = \lambda \frac{dx_L}{dc_H} + (1-\lambda) \frac{dx_H}{dc_H}$, I get:

$$\frac{dx_H}{dc_H} = -\frac{C_{xc} k_H + \lambda C_x \frac{2\tau_\theta}{\rho^2(1-t)^2}}{C_{xx} k_H + C_x \left[ 1+ (1-\lambda) \frac{2\tau_\theta}{\rho^2(1-t)^2} \right]} \frac{dx_L}{dc_H}$$

Similar result can be derived for:

$$\frac{dx_L}{dc_H} = -\frac{(1-\lambda) C_x \frac{2\tau_\theta}{\rho^2(1-t)^2}}{C_{xx} k_L + C_x \left[ 1+ \lambda \frac{2\tau_\theta}{\rho^2(1-t)^2} \right]} \frac{dx_H}{dc_H}$$

Substituting the last equation in the previous one, I derive the reduced form of $\frac{dx_H}{dc_H}$:

$$\frac{dx_H}{dc_H} = -\frac{\left( C_{xx} k_H + C_x \left[ 1+ \lambda \frac{2\tau_\theta}{\rho^2(1-t)^2} \right] \right) C_{xc} k_H}{C_{xx} k_H + C_x \left[ k_L + k_H + \frac{2\tau_\theta}{\rho^2(1-t)^2} K \right] + C_x \left[ 1+ \frac{2\tau_\theta}{\rho^2(1-t)^2} \right]} < 0$$

and, similarly, the reduced form of $\frac{dx_L}{dc_H}$:

$$\frac{dx_L}{dc_H} = \frac{\left[ (1-\lambda) C_x \frac{2\tau_\theta}{\rho^2(1-t)^2} \right] C_{xc} k_H}{C_{xx} k_H + C_x \left[ k_L + k_H + \frac{2\tau_\theta}{\rho^2(1-t)^2} K \right] + C_x \left[ 1+ \frac{2\tau_\theta}{\rho^2(1-t)^2} \right]} > 0$$

The impact of $c_h$ on the aggregate informativeness is always negative as long as the assumptions on the cost function $C(x,c)$ hold:

$$\frac{dl}{dc_H} < 0 \Leftrightarrow \frac{dx_L/dc_H}{dx_H/dc_H} < -\frac{1-\lambda}{\lambda}$$

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The previous results holds for all $c_H < \bar{c}(\Theta)$. While for all $c_H > \bar{c}(\Theta)$,
\[
\frac{dx_H}{dc_H} = \frac{dx_L}{dc_H} = \frac{dI}{dc_H} = 0
\]

Table 1: Optimal information choices and the endogenous threshold. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\lambda = 0.25$, $c_L = 0.01$, $c_H = 0.1$, $t = 0$

<table>
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<tr>
<th>$c_H$</th>
<th>$x_H$</th>
<th>$I$</th>
<th>$x_L$</th>
<th>$c_H - \bar{c}$</th>
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<tbody>
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Table 2: Optimal information choices and the endogenous threshold. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $\lambda = 0.25$, $c_L = 0.01$, $c_H = 0.1$, $t = 0$

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<th>$x_L$</th>
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Market Price Variance

Market price function is \( pR = a + b\pi - d\theta \) and the market price variance is:

\[
\sigma^2_{pR} = b^2 \frac{1}{\tau_\pi} + d^2 \frac{1}{\tau_\theta}
\]

Deriving with respect to the aggregate informativeness holds:

\[
\frac{d\sigma^2_{pR}}{dI} = \frac{2b}{\tau_\pi} \frac{db}{dI} + \frac{2d}{\tau_\theta} \frac{dd}{dI}
\]

with

\[
\frac{db}{dI} = \frac{dK}{dI} \frac{(1-b)}{K}, \quad \frac{dd}{dI} = \frac{1}{K} \left( \frac{\tau_\theta}{\rho(1-t)} - \frac{dK}{dI} \right), \quad \frac{dK}{dI} = 1 + 2 \frac{I_\theta}{\rho^2(1-t)^2}
\]

and substituting back into the previous formula, I get:

\[
\frac{d\sigma^2_{pR}}{dI} = 2 \left[ \left( \frac{b}{\tau_\pi} \frac{dK}{dI} \frac{1}{K} + \frac{d}{\rho(1-t)} \frac{1}{K} \right) - \frac{dK}{dI} \frac{1}{K} \sigma^2_{pR} \right]
\]

The derivative of the market price variance with respect to the aggregate informativeness is negative when the market price variance is greater than the sum of the prior variance multiplied by the sensitivity of the market price to the asset payoff shocks and the variance of the noisy supply multiplied by the sensitivity of the market price to the noisy supply shocks and the inverse of the marginal impact of the aggregate informativeness on the precision of the average agent’s variance:

\[
\frac{d\sigma^2_{pR}}{dI} < 0 \iff \sigma^2_{pR} > \frac{b}{\tau_\pi} + \frac{d}{\rho(1-t)} \frac{1}{dK/dI}
\]

Table 3: Market price variance. \( R = 1 \), \( W_1 = 1 \), \( \rho = 1 \), \( \mu_\pi = 1 \), \( \mu_\theta = 0 \), \( \tau_\pi = 1 \), \( \tau_\theta = 1 \), \( \lambda = 0.25 \), \( c_L = 0.01 \), \( c_H = 0.03 \), \( t = 0 \)

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Table 4: Market price variance. $R = 1$, $W_1 = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$, $c_H = 0.03$, $t = 0$.

<table>
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<th>$\tau_\pi = \tau_\theta = 0.5$</th>
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Table 5: Market price variance. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$, $t = 0$.

<table>
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<tr>
<th>$c_H$</th>
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Figure 2: The market price variance (continuous line) and the relative threshold (dashed line). The inequality in the financial information costs is computed keeping fixed $c_L$ and letting $c_H$ increase. The four panels refer to different scenarios in the fundamentals’ uncertainty: from high uncertainty ($\tau_\pi = \tau_\theta = 0.3$) to low uncertainty ($\tau_\pi = \tau_\theta = 2$). $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\lambda = 0.25$, $c_L = 0.01$, $t = 0$. 

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Table 6: Market price variance. $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $c_L = 0.01$, $c_H = 0.03$, $t = 0$.

<table>
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</table>

Appendix C

I run Monte Carlo simulations to compute the tax rate that satisfies the policy makers’ budget constraint: the expected fiscal revenue ($EFR$) is equal to the cost of the policy. The former is given by:

$$EFR = E[t \int j \max\{\alpha_j^*(W_j - C_j^*) \frac{\pi - pR}{p}, 0\} dj]$$

The condition according to which the individual capital gain is greater than zero is $\alpha_j^* \frac{(\pi - pR)}{p} > 0$. Thus, substituting $\alpha_j^*$ with (5) and simplifying, I derive the following:

$$EFR = E[\frac{t}{p(1-t)}(\pi - pR) \int j \alpha_j^*(E[\pi | s_j, p] - pR) dj] \text{ if } \alpha_j^* \frac{(\pi - pR)}{p} > 0$$

In order to write the code for the Monte Carlo simulation, I distinguish between informed and uninformed agents. To simplify the notation, I drop the subscript about the type.\footnote{The Matlab code is available on request.}

For the informed agents, the condition of positive capital gains can be rewritten in terms of the private signal: conditional to the realization of the fundamentals, the private signal leads to capital gains or capital losses. I derive a threshold according to which it is possible to distinguish the traders who get capital gains and to compute these amounts. Thus, the condition $\alpha_j^* \frac{(\pi - pR)}{p} > 0$ holds when:

$$s > \overline{s} \text{ if } \frac{p(\pi - pR)}{p} > 0$$
$$s < \overline{s} \text{ if } \frac{p(\pi - pR)}{p} < 0$$

where $\overline{s} = (pRk - \mu_\pi \tau_\pi - \xi \tau_\theta)/x$.

Thus, I can write:

$$\int j \alpha_j^*(E[\pi | s_j, p] - pR) dj = \left\{ \begin{array}{ll} k \int_{s > \overline{s}} E[\pi | s_j, p] - pR \ dF(s) & \text{if } \frac{p(\pi - pR)}{p} > 0 \\ k \int_{s < \overline{s}} E[\pi | s_j, p] - pR \ dF(s) & \text{if } \frac{p(\pi - pR)}{p} < 0 \end{array} \right.$$
and, given the properties of the truncated normal distribution, I can derive a formula that depends only on the fundamentals $\pi$ and $\theta$:

$$k \int_{s_>s} E[\pi|s_j, p] - pR dF(s) = \pi + \sqrt{\pi/n} \eta((\pi-\pi)\sqrt{x}) + (\mu_{\pi} \tau_{\pi} + \xi \xi - pRk)(1 - F(\pi))$$

where $\eta((\pi-\pi)\sqrt{x}) = 1 - \Phi((\pi-\pi)\sqrt{x})$. Similarly, for the other case.

For the uninformed agents ($x = 0$), the condition of positive capital gains can be rewritten in terms of the public signal. Thus, the condition $\alpha_j(\pi-pR)/p > 0$ holds when:

$$\xi > \bar{\xi} \text{ if } \frac{p(\pi-pR)}{p} > 0$$

$$\xi < \bar{\xi} \text{ if } \frac{p(\pi-pR)}{p} < 0$$

where $\bar{\xi} = (pRk - \mu_{\pi} \tau_{\pi})/\tau_{\xi}$.

Thus, I can write:

$$\int k_j^*(E[\pi|p] - pR) dj = \begin{cases} k(E[\pi|p] - pR) & \text{if } \frac{p(\pi-pR)}{p} > 0 \quad \vee \quad \xi > \bar{\xi} \\ k(E[\pi|p] - pR) & \text{if } \frac{p(\pi-pR)}{p} < 0 \quad \vee \quad \xi < \bar{\xi} \\ 0 & \text{otherwise} \end{cases}$$

The budget constraint for the policy makers when both types are informed and $\frac{p(\pi-pR)}{p} > 0$ is the following:

$$\frac{\rho}{\rho(1-\gamma)} E \left[ (\pi-pR) \left\{ \lambda \left[ x_L \left( \pi + \sqrt{\pi/n} \eta((\pi_L-\pi)\sqrt{x_L}) \right) + (\mu_{\pi} \tau_{\pi} + \tau_{\xi} \xi - pRk_L)(1 - F_L(\pi_L)) \right] \\
+ (1 - \lambda) \left[ x_H \left( \pi + \sqrt{\pi/n} \eta((\pi_H-\pi)\sqrt{x_H}) \right) + (\mu_{\pi} \tau_{\pi} + \tau_{\xi} \xi - pRk_H)(1 - F_H(\pi_H)) \right] \right\} \right] = \lambda$$

Similarly for the other cases.

Table 7: Tax rate: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_{\pi} = 1$, $\mu_{\theta} = 0$, $\tau_{\pi} = 1$, $\tau_{\theta} = 1$, $\lambda = 0.25$, $c_H = 0.1$.

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</tbody>
</table>

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Table 8: Tax rate: $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$, $c_L = 0.01$, $c_H = 0.1$.

<table>
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Figure 3: Aggregate Welfare (continuous line). The inequality in the financial information costs is computed keeping fixed $c_H$ and letting $c_L$ decrease. The two panels refer to different policies: the first improves the ability of the literates (with $\lambda = .25$), the second increases the fraction of literates (with $c_L = 0.01$). $R = 1$, $W_1 = 1$, $\rho = 1$, $\mu_\pi = 1$, $\mu_\theta = 0$, $\tau_\pi = 1$, $\tau_\theta = 1$. 

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References


