Working Paper 142/14

EQUALLY-WEIGHTED RISK CONTRIBUTION PORTFOLIOS: AN EMPIRICAL STUDY USING EXPECTED SHORTFALL

Elisabetta Cagna
Giulio Casuccio

Equally-weighted Risk Contribution Portfolios: an empirical study using expected shortfall

Elisabetta Cagna
Giulio Casuccio[1]

Abstract

The high volatility observed in financial markets during the last crisis prompted renewed interest in designing truly diversified portfolios. One of the most interesting approach proposed by recent literature is the Equally-weighted Risk Contribution strategy (Maillard et al., 2009), usually implemented with standard deviation as risk measure: our paper extends this approach introducing expected shortfall. The expected shortfall risk contributions are computed through a non-parametric approach which aims to reduce the estimation error generated by the historical sample applying a bootstrap resampling procedure. The ex-post performance analysis also accounts for realistic transaction costs. We find superiority of the ERC portfolios, with better Sharpe ratio along with asymmetric performance metrics.

Keywords: Equal Risk Contribution, Risk Parity, Expected Shortfall, Bootstrap, Portfolio Construction.

1 Introduction

In the last years financial markets have been characterized by high volatility and the issue regarding how to create truly diversified portfolios is presented as a challenging problem to be solved. Markowitz in 1952 [14] proposed the Mean Variance model to choose an optimal mix of securities and achieve a trade-off between risk and return. Some drawbacks make this strategy

[1]Elisabetta Cagna, elisabetta.cagna@hotmail.it, Junior Risk Manager, Symphonia Sgr, Via Gramsci 7, 10121, Turin, Italy. Giulio Casuccio, giulio.casuccio@fondacosgr.it, Head of Fixed Income and Multi-Asset Investments, Fondaco Sgr, Corso Vittorio Emanuele II, 71, 10128, Torino, Italy.
We thank Francesco Cesarone for having contributed to point out some further structural aspects of the research and the seminars participants at XV Workshop on Quantitative Finance, Department of Economics and Management, Florence. We are also greatful to Fabio Bagliano, Giovanna Nicodano and Luisa Tibiletti for suggestions and discussion and the seminars participants at Workshop in Capital Markets, CERP-Collegio Carlo Alberto and CINTIA.

not commonly used in practice. Mistakes in estimating the returns and the variance-covariance matrix, inputs in the optimization problem, can alter the performance of the portfolios and erode the gains of the strategy [2].

An alternative approach is the Equally Weighted strategy (De Miguel in [9]), also EW, consisting in equally splitting the wealth among different securitues. The EW portfolios outperform many other quantitative complicated models with higher Sharpe Ratio and Certainty Equivalent return for several empirical datasets. The downside is an unbalanced portfolio in terms of risk. Consider an equally weighted portfolio in bond and stock: the assets have the same weight in the portfolio but the equity component bears a higher risk than the bond component. In conclusion, the EW model provides diversification only in terms of capital.

In 2009 Maillard, Roncalli and Teiletche [15] presented the Equally-weighted Risk Contribution strategy, also Risk Parity or ERC. Each portfolio component contributes to the same extent to the overall risk, resulting in a portfolio truly diversified in terms of risk. The most difficult choice consists in selecting the proper risk measure to assess the risk contributions. Many authors have used the standard deviation, such as Maillard et al. in 2009 [15], Linzmeier in 2011 [13] and Stefanovits in 2010 [20], defining the standard deviation risk contribution of asset i as the share of portfolio's overall risk attributable to that component. The risk contribution can be defined also as a combination of both the performance contribution and the standard deviation risk contribution of each asset (Roncalli [19]).

From a theoretical point of view the necessary condition that a risk measure has to satisfy is the linear homogeneity in the weights: only in this case the total risk of the portfolio can be fully decomposed into different components. The standard deviation as well as the expected shortfall (ES) satisfy this requirement. Stefanovits in [20] and Colucci in [8] implemented the ERC strategy with the latter one. Unlike the standard deviation, the ES is an asymmetrical and coherent (Artzner et al. [4]) risk measure and it does not require any assumptions on the probability distribution of asset returns. The approach of Stefanovits has been considered as starting point of this paper with the introduction of the bootstrap to generate estimates less related to the market trends.

The rest of the article is organized as follows. After a brief review of the literature about the Equally-weighted Risk Contribution strategy in section

[2]For further details, see De Miguel V. et al., 2009, Optimal Versus Naive Diversification: How Inefficient is the $1/N$ Portfolio Strategy?, The Review of Financial Studies, **22** , No. 5, 1915-1953.

[2], section [3] describes the methodology to estimate the risk contributions and to define the ERC portfolios. Section [4] presents the empirical results of the analysis. The ERC portfolios are constructed by using twelve different financial instruments from 01/01/1999 to 31/12/2011 and the descriptive statistics are compared with the ones of the EW. Finally, a summary of the paper with the main highlights and some further observations is provided.

2 Review of the literature

Maillard et al. in [15] provided for the first time in the literature the idea of Equally-weighted Risk Contribution (ERC) allocation, defining the standard deviation risk contribution of each asset in the portfolio as the share of the portfolio standard deviation attributable to each component. The results of the backtest show lower performance of the Equally Weighted portfolios[3] than the ERC ones, while Minimum Variance portfolios[4] present higher Sharpe Ratios but higher drawdowns in the short-run. Similar results are obtained by Linzmeier in [13] and Stefanovits in [20]. One of latest researches about Risk Parity (Roncalli [19]) has defined the standard deviation risk contribution of asset i as the weighted average of its return contribution and its risk contribution. The suggested definition of risk contribution is flexible enough to encompass the use of alternative risk measures as the expected shortfall.

The literature offers several variants to the original approach of Maillard et al. [15], mainly regarding the possibility of computing the contributions with asymmetric risk measures. From a theoretical point of view the necessary condition that a risk measure has to satisfy is the linear homogeneity in the weights. In this case, the total risk of the portfolio can be fully decomposed into different components. Stefanovits in [20] proves that in the particular case of multivariate normal distribution with 0-mean and unit variance, it is possible to derive the partial derivative of the ES with respect to the weight. Worthy of attention is the proposal of Colucci [8] to simplify the construction of the ERC portfolios by exploiting the convexity property of expected shortfall. The risk of the portfolio is almost equal to its maximum risk and the absolute contribution to the maximum risk is the product between the allocation in each asset and its expected shortfall.

[3]For further details, see De Miguel V., Garlappi L. and Uppal R., 2009, Optimal Versus Naive Diversification: How Inefficient is the $1/N$ Portfolio Strategy?, The Review of Financial Studies, **22**, No. 5, 1915-1953.

[4] Markowitz H., 1952, Portfolio selection, Journal of Finance, **7**, 77-91.

Regarding the problem of estimating the risk contributions, and in particular the ES risk contributions, suggestions from the literature involve either simulation by Monte Carlo method as suggested by Muromachi in [16], estimation using historical data as suggested by Stefanovits in[20] or with the Filtered Bootstrap approach as Colucci in [8] to capture the not-Gaussian distribution of the risk factors.

3 Equally weighted shortfall contributions

Consider a portfolio of N risky assets with vector of weights $\boldsymbol{w}$ and identify with $\rho(\boldsymbol{w})$ the total risk of the portfolio. The Equally-weighted Risk Contribution strategy aims to construct portfolios such that the risk contribution of each asset to the total risk of the portfolio is the same[5], i.e.

$$RC_i^{\rho}(\boldsymbol{w}) = w_i \frac{\delta \rho(\boldsymbol{w})}{\delta\ w_i} = w_j \frac{\delta \rho(\boldsymbol{w})}{\delta\ w_j} = RC_j^{\rho}(\boldsymbol{w}) \tag{1}$$

for $i = 1, \dots, N$ and $j = 1, \dots, N$. The risk contribution is defined as the product of weight in asset i times the first derivative of the portfolio risk with respect to the weight allocated in asset i. The ERC criterion can be applied with any kind of risk measure that satisfies the necessary condition of linear-homogeneity[6] in the weights. Moreover, the risk contributions must be additive in order to sum up to the total risk of the portfolio.

Most of the literature has chosen the standard deviation for the implementation. One of its main drawbacks is the underlying hypothesis of Gaussian distribution of returns. Due to the empirical evidence which rejects this assumption, the use of tail-risk measures, and in particular of the expected shortfall (ES), turns out to be a reasonable choice. The ES of a portfolio is a measure of the expected portfolio return at each quantile of its distribution.

Let Y be the profit (or loss) of a portfolio on a particular time horizon T and let $\alpha \in (0,1)$ be the confidence level. Assuming $E\left[Y^-\right] < \infty$, the tail-mean at level α is defined as [7]

$$\bar{y_\alpha} = TM_{(\alpha)}(Y) = \alpha^{-1}\left(E\left[Y \mathbb{1}_{Y \le y_\alpha}\right] + \left(y_\alpha \alpha - P\left[Y \le y_\alpha\right]\right)\right) \tag{2}$$

[5] Stefanovits D., 2010, Equal Contributions to Risk and Portfolio Construction, Master Thesis, ETH Zurich.

[6] For further details about the definition of homogeneity, see Tasche D., 2000, Conditional Expectation as Quantile Derivative, Department of Mathematics, TU-Munchen.

[7] The definition is originally proposed by Acerbi et al. in [1] and [2]; they also provide a proof to demonstrate the sub-additivity property of the expected shortfall.

The expected shortfall (ES) at level α is

$$ES_{(\alpha)}(Y) = -\bar{y_\alpha} \tag{3}$$

The sensitivity to small changes in the confidence level of the tail-risk measures could be a problem because of the not continuity in the confidence level. On the contrary, the ES does not suffer any consequences due to these changes and, if correctly estimated, it represents an important and useful tool in the evaluation of the risk with no restrictions of applicability. After having chosen the most appropriate risk measure for the implementation of the Risk Parity, we consider both the problem of making the first order derivatives of the expected shortfall and computing the partial derivatives. Some specific assumptions on the distribution of the asset returns must be verified to guarantee the existence of the partial derivatives[8]. Under these hypotheses, the expected shortfall evaluated at a certain confidence level α is partially differentiable with partial derivatives given by [9]

$$\frac{\delta ES_{(\alpha)}}{\delta w_i}(\boldsymbol{w}) = -\frac{1}{\alpha}\left\{E\left[R_i \mathbb{1}_{\{Y \leq q_\alpha(Y)\}}\right] + E\left[R_i \mid \; Y = q_\alpha(Y)\right](\alpha - P\left[Y \leq q_\alpha(Y)\right])\right\} \tag{4}$$

where R_i is the return of asset i and $q_\alpha(Y)$ is the quantile of the distribution of Y at the confidence level α. Stefanovits in [20] proves that, in case of a multivariate normal distribution $\frac{\boldsymbol{Y}-\boldsymbol{w'\mu}}{\sqrt{\boldsymbol{w'\Sigma w}}}$, where $\boldsymbol{w'\mu}$ is the mean and $\boldsymbol{w'\Sigma w}$ the variance of the distribution, the expected shortfall is

$$\begin{aligned}
ES_{(\alpha)}(\boldsymbol{w}) &= -E\left[Y \mid \; Y \leq VaR_{(\alpha)}(Y)\right] \\
&= -\boldsymbol{w'\mu} - \sqrt{\boldsymbol{w'\Sigma w}} E\left[\frac{\boldsymbol{Y}-\boldsymbol{w'\mu}}{\sqrt{\boldsymbol{w'\Sigma w}}}\middle| \frac{\boldsymbol{Y}-\boldsymbol{w'\mu}}{\sqrt{\boldsymbol{w'\Sigma w}}} \leq -VaR_{(\alpha)}\left(\frac{\boldsymbol{Y}-\boldsymbol{w'\mu}}{\sqrt{\boldsymbol{w'\Sigma w}}}\right)\right] \\
&= -\boldsymbol{w'\mu} - \frac{\sqrt{\boldsymbol{w'\Sigma w}}}{\alpha}\int_{-\infty}^{\Phi^{-1}(\alpha)} x\phi(x)dx \\
&= -\boldsymbol{w'\mu} - \frac{\sqrt{\boldsymbol{w'\Sigma w}}}{\alpha}\phi(\Phi^{-1}(\alpha))
\end{aligned} \tag{5}$$

[8] For simplicity, it is supposed that all these assumptions are satisfied.

[9] For mathematical details see Tasche D., 1999, Conditional Expectation as Quantile Derivative,Department of Mathematics, TU-Munchen and Tasche D., 2000, Risk contribution and performance measurement, Department of Mathematics, TU-Munchen. The author, after having introduced the necessary assumptions for making the quantile function partially differentiable, provided a list of possible situations in which they are satisfied.

and its partial derivatives can be computed as

$$\frac{\delta ES_{(\alpha)}}{\delta w_i}(\boldsymbol{w}) = \frac{(\boldsymbol{\Sigma w})_i}{\alpha\sqrt{\boldsymbol{w}'\boldsymbol{\Sigma w}}}\phi(\Phi^{-1}(\alpha)) - \mu_i \tag{6}$$

The risk contribution of asset i to the expected shortfall of a portfolio is defined as

$$\begin{aligned} RC_i^{ES_\alpha}(\boldsymbol{w}) = -w_i\frac{1}{\alpha}\left\{E\left[R_i\mathbb{1}_{\{\boldsymbol{w}'\boldsymbol{R}\leq q_\alpha(\boldsymbol{w}'\boldsymbol{R})\}}\right]\right. \\ \left. + E\left[R_i\middle|\boldsymbol{w}'\boldsymbol{R} = q_\alpha(\boldsymbol{w}'\boldsymbol{R})\right]\left(\alpha - P\left[\boldsymbol{w}'\boldsymbol{R}\leq q_\alpha(\boldsymbol{w}'\boldsymbol{R})\right]\right)\right. \end{aligned} \tag{7}$$

where the vector $\boldsymbol{R}$ contains the asset returns and $\boldsymbol{w}$ the portfolio weights, with $i = 1, \ldots, N$. In case of a continuous profit distribution it simplifies to

$$RC_i^{ES_\alpha}(\boldsymbol{w}) = -w_i E\left[R_i \mid \ \boldsymbol{w}'\boldsymbol{R} \leq VaR_\alpha(\boldsymbol{w})\right] \tag{8}$$

where $VaR_\alpha(\boldsymbol{w})$ is the Value-at-Risk of the portfolio at the confidence level α.

Once the estimator for (8) is available, it is possible to compute the risk contributions of the expected shortfall. Approaches for assessing the ES risk contributions consist in either simulation by either Monte Carlo method or other methods as suggested by Muromachi in [16], estimation with the Filtered Bootstrap approach as Colucci in [8] or using historical data as proposed by Stefanovits in [20]. In this context, the latter approach is followed, combined with the use of bootstrap. Dealing with the time series of returns, the high dependence of the estimates on the market trends could imply distorted results. The bootstrap approach allows to overcome this problem reducing the impact of estimation errors. The starting vector of portfolio weights used at this stage is equally-weighted since a sensitivity analysis has confirmed the stability of the results obtained in this way. The portfolio returns are sorted and, on the basis of these sorted returns, also the simulated returns of the individual assets in the portfolios are sorted. The ES risk contributions are computed by summing the first αn values of the sorted simulated returns of the individual component, where α is the confidence level and n is the total number of simulated returns, and multiplying the sum by $\left(-\frac{1}{\lfloor\alpha n\rfloor}\right)$. The output is a vector containing the contribution of each asset in the portfolio to the total risk. In order to construct an ERC portfolio, each of these contributions must be equal and a SQP (Sequential

Equity	Fixed Income	Short-term Interest Rate	Commodity
S&P 500	10 Y USA	Euribor 3-Months	Crude oil
Euro STOXX 50	5 Y USA	Fed Funds Rate	Gold
	2 Y USA		
	10 Y DE		
	5 Y DE		
	2 Y DE		

Table 1: Portfolio components.

Quadratic Programming) algorithm is used to find the optimal solution. The objective function in the optimization is

$$f(\boldsymbol{w}) = \sum_{i=1}^{N}\sum_{j=1}^{N}(RC_i^{ES_\alpha}(\boldsymbol{w}) - RC_j^{ES_\alpha}(\boldsymbol{w}))^2 \tag{9}$$

where N is the total number of assets in the portfolio. The aim consists in minimizing the square difference between the expected shortfall's risk contribution of asset i and the expected shortfall's risk contribution of asset j times the weights in the respective assets. The implementation of the ERC strategy is done using Matlab 7.5.0 with function *fmincon* which seeks to minimize the above function under some specific constraints, in particular the impossibility of short selling and the possibility to have leveraged positions.

4 Empirical study

The investable universe of th ERC strategy consists of four asset calsses (Equity, Fixed Income, Short-term Interest Rate and Commodity), each divided into sub-classes as showed in Table 1. With the purpose of achieving a higher liquidity and flexibility of the simulated portfolios, only exchange-traded futures are considered in the implementation. The futures contracts are treated daily and they are characterized by a high liquidity, involving low transaction costs[10]. Moreover, it is not necessary to actually invest the whole amount of money the investor wants to allocate because the operations in futures are characterized by the margin system. According to this proceeding, the subscriber must deposit only a percentage of the total invested amount, called margin, by means of a clearing member as a guarantee at

[10]For this reason, transaction costs are assumed to generate a proportional cost of one basis point.

	Ann. Ret.	Comp. Ret.	Ann. Std. Dev.	Sharpe Ratio	Ann. Turnover	Max DD	Ann. Down. Vol.
EW 100% lev.	3.67%	54.02%	4.74%	0.7736	2.41%	-18.18%	2.99%
ERC 150% lev.	3.80%	56.49%	2.78%	1.3672	49.57%	-4.73%	1.28%

Table 2: Descriptive statistics for the Equally Weighted portfolios with no leveraged positions and the Equally-weighted Risk Contribution portfolios with leveraged positions at level 150% (annualized return, compounded return, annualized standard deviation, Sharpe ratio, average annualized turnover, maximum drawdown and annualized downside volatility).

	Ann. Ret.	Comp. Ret.	Ann. Std. Dev.	Sharpe Ratio	Max DD	Ann. Down. Vol.
EW 100% lev.	3.66%	53.97%	4.74%	0.7729	-18.18%	3.00%
ERC 150% lev.	3.74%	53.39%	2.778%	1.3452	-4.74%	1.29%

Table 3: Net descriptive statistics for the Equally Weighted portfolios with no leveraged positions and the Equally-weighted Risk Contribution portfolios with leveraged positions at level 150%.

maturity. The difference between the total amount and the margin amount remains in the investor's portfolio and it can be allocated, for example, in a risk-free asset. In this research no further allocations are made; anyway, if an adjunctive investment in a risk-free asset were made, it would enhance the returns of the strategies. Finally, a strategy which invests in futures is defined to be partially funded and it is not necessary to borrow money to leverage the portfolio position.
The sample period goes from 01/01/1999 to 31/12/2011 with weekly prices, for a total of 676 historical observations. The backtests are built by using a rolling-window of two years and by rebalancing the portfolios every twenty working days[11]. The Equally Weighted strategy has been used as benchmark for the performance of the ERC. The EW portfolio permits to diversify the portfolio by equally splitting the total wealth among all the securities providing a diversification in terms of capital. The performance results of the EW obtained without imposing any level of leverage is set as either a target return or a target risk and then a certain level of leverage is introduced in the optimization process to allow the ERC portfolios to match respectively either the same performance or the same risk. The cost of leverage is not considered in the analysis.

Table 2 reports the statistics of the EW portfolios when no leverage is imposed and the ERC portfolios with 150% leverage. Despite the similar per-

[11]Since the portfolios are rebalanced with a frequency of 20 working days, the last portfolio is computed at 18/12/2011.

Figure 1: Gross cumulative returns of the Equally Weighted strategy, by imposing constraints on leveraged positions, and the Equally-weighted risk Contribution strategy with a level of leverage of 150%.

formance, the analysis of the annualized standard deviation, the maximum drawdown and the annualized downside volatility demonstrates a lower risk of the ERC portfolios. The standard deviation falls from 4.74% to 2.78%, the maximum drawdown plummets from -18.18% to -4.73% and the annualized downside volatility shows lower concentration of negative returns in the left tail, from 2.99% to 1.28%. Due to the lower standard deviation of the ERC portfolios, the Sharpe ratio is much higher, almost twice. The analysis has been repeated considering the transaction costs since the monthly rebalancing of the ERC strategy involves high turnover. As shown in Table 3, the annualized return is slightly affected as well as the other descriptive statistics. Although both the Sharpe ratios display a reduction, the difference among the two strategies remains meaningful in terms of risk-adjusted performance.

Figure 1 plots the performance in terms of compounded return of both the EW strategy without applying any leverage and the ERC strategy allowing for 150% leverage. Even without analyzing the descriptive statistics and only looking at the graph, it is immediate to capture the higher volatility of the $1/N$ strategy with respect to the risk-balanced one. Figure 2 and Figure 3 display the distributions of the monthly returns. If it is assumed that both the distributions have zero-mean, the ERC strategy avoids the extreme negative results of the EW strategy since its returns are more shifted toward the right side of the distribution.

When the annualized standard deviation of the EW strategy is chosen as target risk, it is necessary to impose a leverage of 250% to enable the ERC strategy to reach similar riskiness. Table 4 presents the descriptive statistics

	Ann. Ret.	Comp. Ret.	Ann. Std. Dev.	Sharpe Ratio	Ann. Turnover	Max DD	Ann. Down. Vol.
EW 100% lev.	3.67%	54.02%	4.74%	0.7736	2.41%	-18.18%	2.99%
ERC 250% lev.	5.48%	89.70%	4.84%	1.1315	99.11%	-9.12%	2.34%

Table 4: Descriptive statistics for the Equally Weighted portfolios with no leveraged positions and the Equally-weighted Risk Contribution portfolios with leveraged positions at level 250% (annualized return, compounded return, annualized standard deviation, Sharpe ratio, annualized average turnover, maximum drawdown and annualized downside volatility).

of both the strategies: the annualized return of the Risk Parity portfolios is 5.48% compared to the 3.67% of the $1/N$ and the maximum drawdown and the annualized downside volatility are respectively -9.12% and 2.34%, rather than -18.18% and 2.99%. The ERC strategy outperforms on a risk- adjusted basis the EW strategy, displaying a Sharpe ratio of 1.1315. Due to the high turnover, the analysis has been repeated considering also the impact of the transaction costs: the annualized standard deviation, the annualized downside volatility and the maximum drawdown remain invariant with respect to the previous case and, despite a lower Sharpe ratio, the risk-balanced portfolios continue to confirm their superiority as shown in Table 5. Even if the ERC strategy with leverage 250% has an average turnover of 99.11%, it is able to get better net results and, in particular, the Sharpe ratio are respectively 1.1056 and 0.7729.

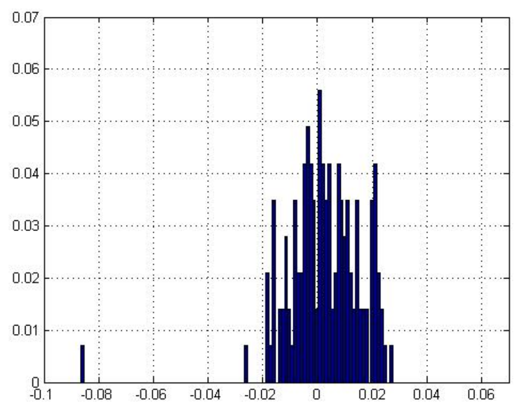

Figure 2: Distribution of the monthly returns of the Equally Weighted portfolios without leverage.

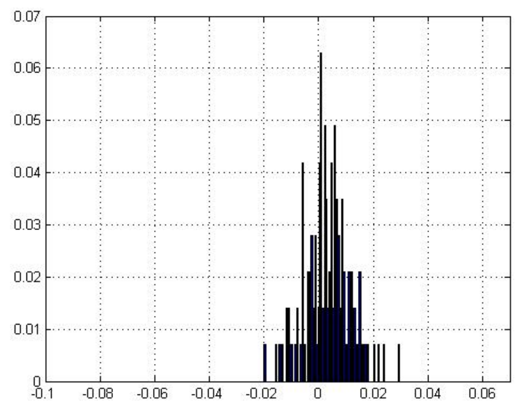

Figure 3: Distribution of the monthly returns of the Equally-weighted Risk Contribution portfolios with 150% leverage.

	Ann. Ret.	Comp. Ret.	Ann. Std. Dev.	Sharpe Ratio	Max DD	Ann. Down. Vol.
EW 100% lev.	3.66%	53.97%	4.74%	0.7729	-18.18%	3.00%
ERC 250% lev.	5.36%	89.05%	4.84%	1.1056	-9.07%	2.35%

Table 5: Net descriptive statistics for the Equally Weighted portfolios with no leveraged positions and the Equally-weighted Risk Contribution portfolios with leveraged positions at level 250%.

Figure 4 compares the performance of the two strategies, while the distributions of the weekly returns, respectively of the EW strategy without leveraged positions and the ERC strategy with 250% leveraged positions, are plotted in Figure 5 and Figure 6: the Risky Parity portfolios are not characterized by the extreme negative observations of the EW portfolios and, moreover, their returns are much more concentrated on the right side of the distribution.

In Figure 7 are plotted the weights of the ERC strategy implemented with 150% leverage clustered in four asset classes, i.e Equity, Fixed Income, Short Term Interest Rate and Commodity. By definition the Risk Parity approach allows to increase the investment in less risky assets to the detriment of the allocation in riskier ones. In particular, the optimal allocation to equity has become zero in periods of high distress (2008-2009). In these periods the portfolios did not suffer the main drawbacks which in contrary led the EW portfolios to significant portfolio losses. The majority of the portfolio remains invested in Fixed Income, while lower exposure to Equity

Figure 4: Gross cumulative returns of the Equally Weighted strategy imposing constraints on leveraged positions and the Equally-weighted rick Contribution strategy with a level of leverage of 250%.

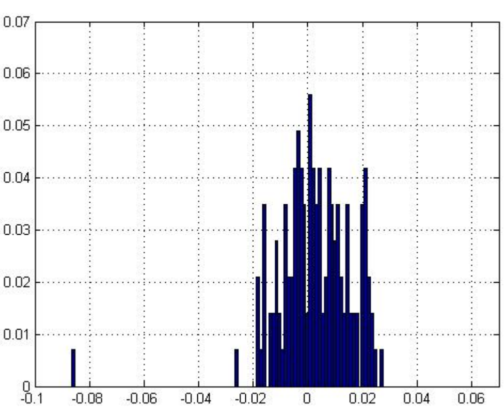

Figure 5: Distribution of the monthly returns of the Equally Weighted portfolios without leverage.

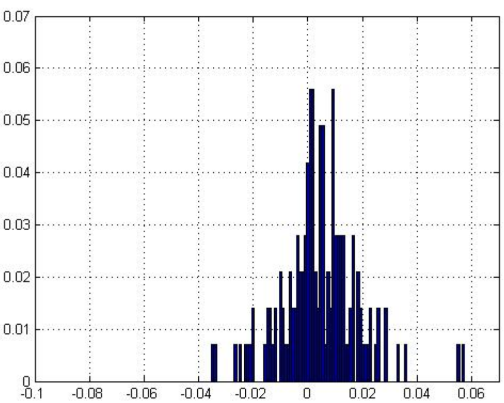

Figure 6: Distribution of the monthly returns of the Equally-weighted Risk Contribution portfolios with 250% leverage.

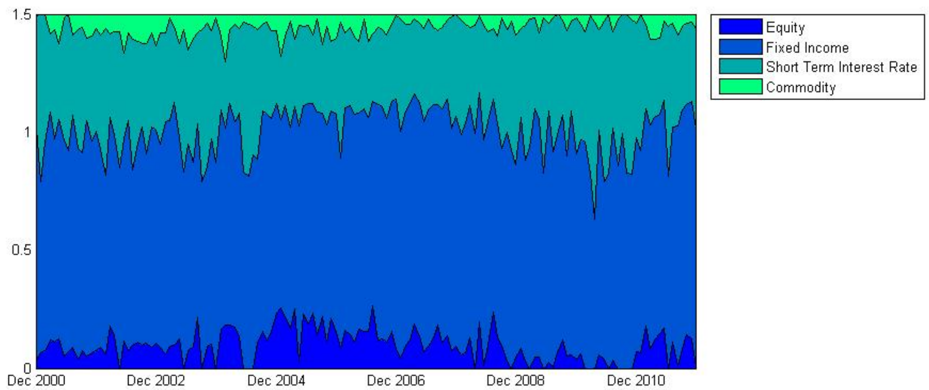

Figure 7: Evolution of the optimal weights over time of the ERC portfolios with 150% leverage.

and Commodity. In Figure 8 are plotted the weights of the ERC strategy implemented with 250% leverage clustered in the same asset classes, showing similar characteristics observed for the 150% case with a slightly higher variability.

The difference in the maximum drawdown and the downside volatility between the ERC and the EW portfolios in both the proposed case is coherent with the choice of the expected shortfall as risk measure, which focuses on

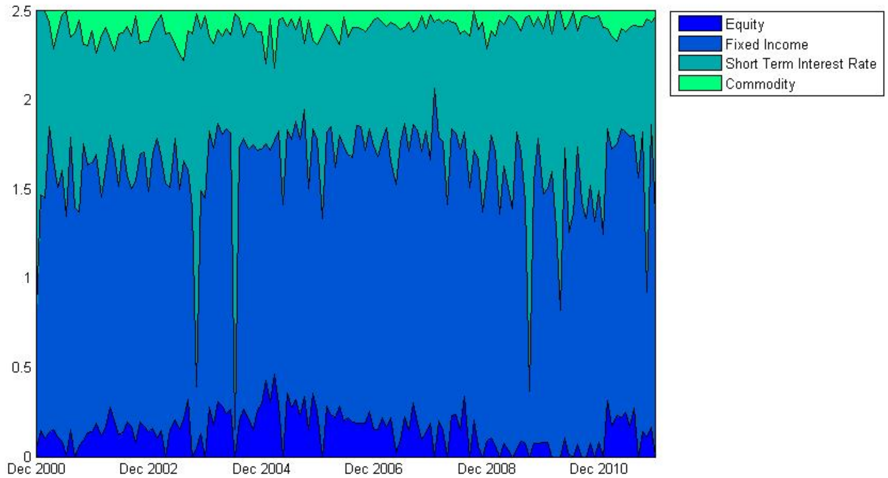

Figure 8: Evolution of the optimal weights over time of the ERC portfolios with 250% leverage.

the left side of the return distribution. Therefore, the ES has been a good alternative to the standard deviation because it is able to provide a more suitable measure of the potential negative events. Finally, worthy of attention is the computation of the turnover of the EW strategy: since Maillard in [15] and Stefanovits in [20] have set the turnover of the EW strategy equal to zero, even if the portfolios are rebalanced with constant frequency, their results are inconsistent with the values of the turnover found in this contest. If the turnover were zero, it would mean that the weights before each rebalancing date have remained unchanged with respect to the beginning of the period and that the asset returns during each single period are unchanged; this turnover would not take into account the changes on the financial markets and it would assume invariant weights of the asset classes overall the period, whatever is happening on the markets. On the contrary, in this analysis the turnover has been computed by considering also these further aspects to provide more realistic results: during each month, the returns of the securities change and consequently, at the end of the month, the portfolios are no longer equally weighted.

5 Conclusions

This paper focuses the attention on comparing two asset allocation strategies based on opposite ideas, the capital allocation and the risk allocation: the first one is represented by the Equally Weighted strategy while the second one by the Equally-weighted Risk Contributions strategy. The EW is a naive strategy which consists of splitting equally the wealth among all the available asset classes. On the contrary, the ERC strategy aims to equalize the risk contributions of all the securities to control the total risk of the portfolio.
The implementation of the ERC strategy is much more challenging than the EW's one as it involves both the choice of a risk measure and the estimation of the risk contributions. Regarding the risk measure, we choose the Expected Shortfall, considered more efficient on assessing the riskiness of the portfolios in presence of kurtosis and skewness of the returns. The approach for estimating the risk contributions is the one proposed by Stefanovits in [20] improved with the introduction of the bootstrap. Since the risk contributions are computed on the basis of the simulated returns, they are less affected by historical market trends.
In the empirical simulations the ERC and the EW strategy are compared by setting the annualized return and the annualized standard deviation of the EW portfolios as target return and target risk. In both cases the risk-balanced portfolios have outperformed in terms of Sharpe ratio, annualized volatility and maximum drawdown, but displaying higher turnover. These results are confirmed even considering transaction costs.
The choice of an asymmetrical risk measure seems to be effective as the downside volatility and the maximum drawdown of the ERC portfolios are always lower than the EW ones.
An important contribution of this research to the previous literature consists in the analysis of the net ERC performance which makes the analysis more complete.
Some suggested cues for further analysis consist of comparing directly the approach of Colucci in [8] and Stefanovits in [20] to the one proposed in this setting and to analyze how much better bootstrap turn out to be ex post. Moreover, it could be interesting to compare the results of the implementation both using the standard deviation and the expected shortfall.

References

[1] Acerbi C., Nordio C. and Sirtori C., 2001, Expected Shortfall as a Tool for Financial Risk Management, Working paper.

[2] Acerbi C. and Tasche D., 2001, Expected Shortfall: a natural coherent alternative to Value at Risk, Economic Notes, **31**, 1-10.

[3] Artzner P., Delbaen F., Eber J. and Heath D., 1997, Thinking coherently, Risk, **10**, 68-71.

[4] Artzner P., Delbaen F., Eber J. and Heath D., 1999, Coherent measures of risk, Mathematical Finance, **9**, No. 3, 203-228.

[5] Cliff Asness, Andrea Frazzini and Lasse Heje Pedersen, 2012, Leverage Aversion and Risk Parity, Financial Analysts Journal, **68(1)**, 47-59.

[6] Barone-Adesi G., Giannopoulos K. and Vosper, L. 1999, VaR without correlations for non-linear portfolios, Journal of Futures Markets, **19**, 583-602.

[7] Bertsimas D., Lauprete G.and Samarov A., 2004, Shortfall as a risk measure: properties, optimization and applications, Journal of Economic Dynamics Control, **28**, 1353-1381.

[8] Colucci S., 2011, How Expected Shortfall can simplify the equally-weighted risk contribution portfolio, Working paper.

[9] De Miguel V., Garlappi L. and Uppal R., 2009, Optimal Versus Naive Diversification: How Inefficient is the $1/N$ Portfolio Strategy?, The Review of Financial Studies, **22**, No. 5, 1915-1953.

[10] De Miguel V., Nogales J. and Uppal R., 2009, A Generalized Approach to Portfolio Optimization: Improving Performance By Constraining Portfolio Norms, Management Science, **55**, No. 5, 798-812.

[11] Emmer S., Kratz M. and Tasche D., 2014, What is the best risk measure in practice? A comparison of standard measures, Preprint, Bank of England.

[12] Green, R. and Holli
eld B., 1992, When will mean-variance efficient portfolios be well diversified?, Journal of Financel, **47**, 1785-1809.

[13] Linzmeier D., 2011, Risk Balanced Portfolio Construction, Working paper.

[14] Markowitz H., 1952, Portfolio selection, Journal of Finance, **7**, 77-91.

[15] Maillard S., Roncalli T. and Teiletche J., 2009, On the properties of equallyweighted risk contributions portfolios, Journal of Portfolio Management, **37**, No. 4.

[16] Muromachi Y., 2009, Decomposing total risk of a portfolio into the contributions of individual assets, Graduate School of Social Sciences, Tokyo Metropolitan University.

[17] Qian E., 2006, On the Financial Interpretation of Risk Contribution: Risk Budgets Do Add Up, Journal of Investment Management, **4**, No. 4, 41-51.

[18] Qian E., 2005, Risk Parity Portfolios: Efficient Portfolios through True Diversification, PanAgora Asset Management Paper.

[19] Roncalli T., 2013, Introduction to Risk Parity and Budgeting, Chapman Hall/CRC Financial Mathematics Series.

[20] Stefanovits D., 2010, Equal Contributions to Risk and Portfolio Construction, Master Thesis, ETH Zurich.

[21] Tasche D., 2008, Capital allocation to business units and sub-portfolios: the Euelr principle. In: Resti, A., editor, Pillar II in the New Basel Approach: The Challenge of Economic Capital, Risk Books, 423-452.

[22] Tasche D., 2000, Conditional Expectation as Quantile Derivative, Department of Mathematics, TU-Munchen.

[23] Tasche D., 2002, Expected Shortfall and Beyond, Journal of Banking and Finance, **26(7)**,1519-1533.

[24] Tasche D., 1999, Risk contribution and performance measurement, Department of Mathematics, TU-Munchen.

[25] Ziegel J.F., 2013, Coherence and Elicitability, arXiv preprint, arXiv:1303-1690.

Our papers can be downloaded at:

http://cerp.carloalberto.org/en/publications

CeRP Working Paper Series

N° 142/14	Elisabetta Cagna Giulio Casuccio	Equally-weighted Risk Contribution Portfolios: an empirical study using expected shortfall
N° 141/14	Massimo Baldini Costanza Torricelli Maria Cesira Urzì Brancati	Family ties: occupational responses to cope with a household income shock
N° 140/14	Cecilia Boggio Elsa Fornero Henriette Prast Jose Sanders	Seven Ways to Knit Your Portfolio: Is Investor Communication Neutral?
N° 139/14	Laura Bianchini Margherita Borella	Cognitive Functioning and Retirement in Europe
N° 138/13	Claudio Morana	Insights on the global macro-finance interface: Structural sources of risk factors fluctuations and the cross-section of expected stock returns
N° 137/13	Claudio Morana	New Insights on the US OIS Spreads Term Structure During the Recent Financial Turmoil
N° 136/13	Anna Lo Prete	Inequality and the finance you know: does economic literacy matter?
N° 135/13	Rik Dillingh Henriette Prast Mariacristina Rossi Cesira Urzì Brancati	The psychology and economics of reverse mortgage attitudes: evidence from the Netherlands
N° 134/13	Annamaria Lusardi Olivia S. Mitchell	The Economic Importance of Financial Literacy: Theory and Evidence
N° 133/13	Annamaria Lusardi Pierre-Carl Michaud Olivia S. Mitchell	Optimal Financial Knowledge and Wealth Inequality
N° 132/13	Riccardo Calcagno Sonia Falconieri	Competition and dynamics of takeover contests
N° 131/13	Riccardo Calcagno Maria Cesira Urzì Brancati	Do more financially literate households invest less in housing? Evidence from Italy
N° 130/12	Maela Giofré	Financial Education, Investor Protection and International Portfolio Diversification
N° 129/12	Michele Belloni Rob Alessie Adriaan Kalwij Chiara Marinacci	Lifetime Income and Old Age Mortality Risk in Italy over Two Decades
N° 128/12	Fabio Cesare Bagliano Claudio Morana	Determinants of US Financial Fragility Conditions
N° 127/12	Mariacristina Rossi Serena Trucchi	Liquidity Constraints and Labor Supply
N° 126/11	Margherita Borella Flavia Coda Moscarola Mariacristina Rossi	(Un)expected retirement and the consumption puzzle

N° 125/11	Carolina Fugazza	Tracking the Italian employees' TFR over their working life careers
N° 124/11	Agnese Romiti Mariacristina Rossi	Should we Retire Earlier in order to Look After our Parents? The Role of immigrants
N° 123/11	Elsa Fornero Maria Cristina Rossi Maria Cesira Urzì Brancati	Explaining why, right or wrong, (Italian) households do not like reverse mortgages
N° 122/11	Serena Trucchi	How credit markets affect homeownership: an explanation based on differences between Italian regions
N° 121/11	Elsa Fornero Chiara Monticone Serena Trucchi	The effect of financial literacy on mortgage choices
N° 120/11	Giovanni Mastrobuoni Filippo Taddei	Age Before Beauty? Productivity and Work vs. Seniority and Early Retirement
N° 119/11	Maarten van Rooij Annamaria Lusardi Rob Alessie	Financial Literacy, Retirement Planning, and Household Wealth
N° 118/11	Luca Beltrametti Matteo Della Valle	Does the implicit pension debt mean anything after all?
N° 117/11	Riccardo Calcagno Chiara Monticone	Financial Literacy and the Demand for Financial Advice
N° 116/11	Annamaria Lusardi Daniel Schneider Peter Tufano	Financially Fragile Households: Evidence and Implications
N° 115/11	Adele Atkinson Flore-Anne Messy	Assessing financial literacy in 12 countries: an OECD Pilot Exercise
N° 114/11	Leora Klapper Georgios A. Panos	Financial Literacy and Retirement Planning in View of a Growing Youth Demographic: The Russian Case
N° 113/11	Diana Crossan David Feslier Roger Hurnard	Financial Literacy and Retirement Planning in New Zealand
N° 112/11	Johan Almenberg Jenny Säve-Söderbergh	Financial Literacy and Retirement Planning in Sweden
N° 111/11	Elsa Fornero Chiara Monticone	Financial Literacy and Pension Plan Participation in Italy
N° 110/11	Rob Alessie Maarten Van Rooij Annamaria Lusardi	Financial Literacy, Retirement Preparation and Pension Expectations in the Netherlands
N° 109/11	Tabea Bucher-Koenen Annamaria Lusardi	Financial Literacy and Retirement Planning in Germany
N° 108/11	Shizuka Sekita	Financial Literacy and Retirement Planning in Japan
N° 107/11	Annamaria Lusardi Olivia S. Mitchell	Financial Literacy and Retirement Planning in the United States
N° 106/11	Annamaria Lusardi Olivia S. Mitchell	Financial Literacy Around the World: An Overview
N° 105/11	Agnese Romiti	Immigrants-Natives Complementarities in Production: Evidence from Italy

N° 104/11	Ambrogio Rinaldi	Pension awareness and nation-wide auto-enrolment: the Italian experience
N° 103/10	Fabio Bagliano Claudio Morana	The Great Recession: US dynamics and spillovers to the world economy
N° 102/10	Nuno Cassola Claudio Morana	The 2007-? financial crisis: a money market perspective
N° 101/10	Tetyana Dubovyk	Macroeconomic Aspects of Italian Pension Reforms of 1990s
N° 100/10	Laura Piatti Giuseppe Rocco	L’educazione e la comunicazione previdenziale - Il caso italiano
N° 99/10	Fabio Bagliano Claudio Morana	The effects of US economic and financial crises on euro area convergence
N° 98/10	Annamaria Lusardi Daniel Schneider Peter Tufano	The Economic Crisis and Medical Care Usage
N° 97/10	Carlo Maccheroni Tiziana Barugola	E se l’aspettativa di vita continuasse la sua crescita? Alcune ipotesi per le generazioni italiane 1950-2005
N° 96/10	Riccardo Calcagno Mariacristina Rossi	Portfolio Choice and Precautionary Savings
N° 95/10	Flavia Coda Moscarola Elsa Fornero Mariacristina Rossi	Parents/children “deals”: Inter-Vivos Transfers and Living Proximity
N° 94/10	John A. List Sally Sadoff Mathis Wagner	So you want to run an experiment, now what? Some Simple Rules of Thumb for Optimal Experimental Design
N° 93/10	Mathis Wagner	The Heterogeneous Labor Market Effects of Immigration
N° 92/10	Rob Alessie Michele Belloni	Retirement choices in Italy: what an option value model tells us
N° 91/09	Annamaria Lusardi Olivia S. Mitchell Vilsa Curto	Financial Literacy among the Young: Evidence and Implications for Consumer Policy
N° 90/09	Annamaria Lusardi Olivia S. Mitchell	How Ordinary Consumers Make Complex Economic Decisions: Financial Literacy and Retirement Readiness
N° 89/09	Elena Vigna	Mean-variance inefficiency of CRRA and CARA utility functions for portfolio selection in defined contribution pension schemes
N° 88/09	Maela Giofré	Convergence of EMU Equity Portfolios
N° 87/09	Elsa Fornero Annamaria Lusardi Chiara Monticone	Adequacy of Saving for Old Age in Europe
N° 86/09	Margherita Borella Flavia Coda Moscarola	Microsimulation of Pension Reforms: Behavioural versus Nonbehavioural Approach
N° 85/09	Cathal O’Donoghue John Lennon Stephen Hynes	The Life-Cycle Income Analysis Model (LIAM): A Study of a Flexible Dynamic Microsimulation Modelling Computing Framework
N° 84/09	Luca Spataro	Il sistema previdenziale italiano dallo shock petrolifero del 1973 al Trattato di Maastricht del 1993

N° 83/09	Annamaria Lusardi Peter Tufano	Debt Literacy, Financial Experiences, and Overindebtedness
N° 82/09	Carolina Fugazza Massimo Guidolin Giovanna Nicodano	Time and Risk Diversification in Real Estate Investments: Assessing the Ex Post Economic Value
N° 81/09	Fabio Bagliano Claudio Morana	Permanent and Transitory Dynamics in House Prices and Consumption: Cross-Country Evidence
N° 80/08	Claudio Campanale	Learning, Ambiguity and Life-Cycle Portfolio Allocation
N° 79/08	Annamaria Lusardi	Increasing the Effectiveness of Financial Education in the Workplace
N° 78/08	Margherita Borella Giovanna Segre	Le pensioni dei lavoratori parasubordinati: prospettive dopo un decennio di gestione separata
N° 77/08	Giovanni Guazzarotti Pietro Tommasino	The Annuity Market in an Evolving Pension System: Lessons from Italy
N° 76/08	Riccardo Calcagno Elsa Fornero Mariacristina Rossi	The Effect of House Prices on Household Saving: The Case of Italy
N° 75/08	Harold Alderman Johannes Hoogeveen Mariacristina Rossi	Preschool Nutrition and Subsequent Schooling Attainment: Longitudinal Evidence from Tanzania
N° 74/08	Maela Giofré	Information Asymmetries and Foreign Equity Portfolios: Households versus Financial Investors
N° 73/08	Michele Belloni Rob Alessie	The Importance of Financial Incentives on Retirement Choices: New Evidence for Italy
N° 72/08	Annamaria Lusardi Olivia Mitchell	Planning and Financial Literacy: How Do Women Fare?
N° 71/07	Flavia Coda Moscarola	Women participation and caring decisions: do different institutional frameworks matter? A comparison between Italy and The Netherlands
N° 70/07	Radha Iyengar Giovanni Mastrobuoni	The Political Economy of the Disability Insurance. Theory and Evidence of Gubernatorial Learning from Social Security Administration Monitoring
N° 69/07	Carolina Fugazza Massimo Guidolin Giovanna Nicodano	Investing in Mixed Asset Portfolios: the Ex-Post Performance
N° 68/07	Massimo Guidolin Giovanna Nicodano	Small Caps in International Diversified Portfolios
N° 67/07	Carolina Fugazza Maela Giofré Giovanna Nicodano	International Diversification and Labor Income Risk
N° 66/07	Maarten van Rooij Annamaria Lusardi Rob Alessie	Financial Literacy and Stock Market Participation
N° 65/07	Annamaria Lusardi	Household Saving Behavior: The Role of Literacy, Information and Financial Education Programs (Updated version June 08: “Financial Literacy: An Essential Tool for Informed Consumer Choice?”)

N° 64/07	Carlo Casarosa Luca Spataro	Rate of Growth of Population, Saving and Wealth in the Basic Life-cycle Model when the Household is the Decision Unit
N° 63/07	Claudio Campanale	Life-Cycle Portfolio Choice: The Role of Heterogeneous Under-Diversification
N° 62/07	Margherita Borella Elsa Fornero Mariacristina Rossi	Does Consumption Respond to Predicted Increases in Cash-on-hand Availability? Evidence from the Italian "Severance Pay"
N° 61/07	Irina Kovrova	Effects of the Introduction of a Funded Pillar on the Russian Household Savings: Evidence from the 2002 Pension Reform
N° 60/07	Riccardo Cesari Giuseppe Grande Fabio Panetta	La Previdenza Complementare in Italia: Caratteristiche, Sviluppo e Opportunità per i Lavoratori
N° 59/07	Riccardo Calcagno Roman Kraeussl Chiara Monticone	An Analysis of the Effects of the Severance Pay Reform on Credit to Italian SMEs
N° 58/07	Elisa Luciano Jaap Spreeuw Elena Vigna	Modelling Stochastic Mortality for Dependent Lives
N° 57/07	Giovanni Mastrobuoni Matthew Weinberg	Heterogeneity in Intra-Monthly Consumption. Patterns, Self-Control, and Savings at Retirement
N° 56/07	John A. Turner Satyendra Verma	Why Some Workers Don't Take 401(k) Plan Offers: Inertia versus Economics
N° 55/06	Antonio Abatemarco	On the Measurement of Intra-Generational Lifetime Redistribution in Pension Systems
N° 54/06	Annamaria Lusardi Olivia S. Mitchell	Baby Boomer Retirement Security: The Roles of Planning, Financial Literacy, and Housing Wealth
N° 53/06	Giovanni Mastrobuoni	Labor Supply Effects of the Recent Social Security Benefit Cuts: Empirical Estimates Using Cohort Discontinuities
N° 52/06	Luigi Guiso Tullio Jappelli	Information Acquisition and Portfolio Performance
N° 51/06	Giovanni Mastrobuoni	The Social Security Earnings Test Removal. Money Saved or Money Spent by the Trust Fund?
N° 50/06	Andrea Buffa Chiara Monticone	Do European Pension Reforms Improve the Adequacy of Saving?
N° 49/06	Mariacristina Rossi	Examining the Interaction between Saving and Contributions to Personal Pension Plans. Evidence from the BHPS
N° 48/06	Onorato Castellino Elsa Fornero	Public Policy and the Transition to Private Pension Provision in the United States and Europe
N° 47/06	Michele Belloni Carlo Maccheroni	Actuarial Neutrality when Longevity Increases: An Application to the Italian Pension System
N° 46/05	Annamaria Lusardi Olivia S. Mitchell	Financial Literacy and Planning: Implications for Retirement Wellbeing
N° 45/05	Claudio Campanale	Increasing Returns to Savings and Wealth Inequality
N° 44/05	Henrik Cronqvist	Advertising and Portfolio Choice
N° 43/05	John Beshears James J. Choi David Laibson Brigitte C. Madrian	The Importance of Default Options for Retirement Saving Outcomes: Evidence from the United States

N° 42/05	Margherita Borella Flavia Coda Moscarola	Distributive Properties of Pensions Systems: a Simulation of the Italian Transition from Defined Benefit to Defined Contribution
N° 41/05	Massimo Guidolin Giovanna Nicodano	Small Caps in International Equity Portfolios: The Effects of Variance Risk.
N° 40/05	Carolina Fugazza Massimo Guidolin Giovanna Nicodano	Investing for the Long-Run in European Real Estate. Does Predictability Matter?
N° 39/05	Anna Rita Bacinello	Modelling the Surrender Conditions in Equity-Linked Life Insurance
N° 38/05	Carolina Fugazza Federica Teppa	An Empirical Assessment of the Italian Severance Payment (TFR)
N° 37/04	Jay Ginn	Actuarial Fairness or Social Justice? A Gender Perspective on Redistribution in Pension Systems
N° 36/04	Laurence J. Kotlikoff	Pensions Systems and the Intergenerational Distribution of Resources
N° 35/04	Monika Bütler Olivia Huguenin Federica Teppa	What Triggers Early Retirement. Results from Swiss Pension Funds
N° 34/04	Chourouk Houssi	Le Vieillissement Démographique : Problématique des Régimes de Pension en Tunisie
N° 33/04	Elsa Fornero Carolina Fugazza Giacomo Ponzetto	A Comparative Analysis of the Costs of Italian Individual Pension Plans
N° 32/04	Angelo Marano Paolo Sestito	Older Workers and Pensioners: the Challenge of Ageing on the Italian Public Pension System and Labour Market
N° 31/03	Giacomo Ponzetto	Risk Aversion and the Utility of Annuities
N° 30/03	Bas Arts Elena Vigna	A Switch Criterion for Defined Contribution Pension Schemes
N° 29/02	Marco Taboga	The Realized Equity Premium has been Higher than Expected: Further Evidence
N° 28/02	Luca Spataro	New Tools in Micromodeling Retirement Decisions: Overview and Applications to the Italian Case
N° 27/02	Reinhold Schnabel	Annuities in Germany before and after the Pension Reform of 2001
N° 26/02	E. Philip Davis	Issues in the Regulation of Annuities Markets
N° 25/02	Edmund Cannon Ian Tonks	The Behaviour of UK Annuity Prices from 1972 to the Present
N° 24/02	Laura Ballotta Steven Haberman	Valuation of Guaranteed Annuity Conversion Options
N° 23/02	Ermanno Pitacco	Longevity Risk in Living Benefits
N° 22/02	Chris Soares Mark Warshawsky	Annuity Risk: Volatility and Inflation Exposure in Payments from Immediate Life Annuities
N° 21/02	Olivia S. Mitchell David McCarthy	Annuities for an Ageing World
N° 20/02	Mauro Mastrogiacomo	Dual Retirement in Italy and Expectations

N° 19/02	Paolo Battocchio Francesco Menoncin	Optimal Portfolio Strategies with Stochastic Wage Income and Inflation: The Case of a Defined Contribution Pension Plan
N° 18/02	Francesco Daveri	Labor Taxes and Unemployment: a Survey of the Aggregate Evidence
N° 17/02	Richard Disney and Sarah Smith	The Labour Supply Effect of the Abolition of the Earnings Rule for Older Workers in the United Kingdom
N° 16/01	Estelle James and Xue Song	Annuities Markets Around the World: Money's Worth and Risk Intermediation
N° 15/01	Estelle James	How Can China Solve ist Old Age Security Problem? The Interaction Between Pension, SOE and Financial Market Reform
N° 14/01	Thomas H. Noe	Investor Activism and Financial Market Structure
N° 13/01	Michela Scatigna	Institutional Investors, Corporate Governance and Pension Funds
N° 12/01	Roberta Romano	Less is More: Making Shareholder Activism a Valuable Mechanism of Corporate Governance
N° 11/01	Mara Faccio and Ameziane Lasfer	Institutional Shareholders and Corporate Governance: The Case of UK Pension Funds
N° 10/01	Vincenzo Andrietti and Vincent Hildebrand	Pension Portability and Labour Mobility in the United States. New Evidence from the SIPP Data
N° 9/01	Hans Blommestein	Ageing, Pension Reform, and Financial Market Implications in the OECD Area
N° 8/01	Margherita Borella	Social Security Systems and the Distribution of Income: an Application to the Italian Case
N° 7/01	Margherita Borella	The Error Structure of Earnings: an Analysis on Italian Longitudinal Data
N° 6/01	Flavia Coda Moscarola	The Effects of Immigration Inflows on the Sustainability of the Italian Welfare State
N° 5/01	Vincenzo Andrietti	Occupational Pensions and Interfirm Job Mobility in the European Union. Evidence from the ECHP Survey
N° 4/01	Peter Diamond	Towards an Optimal Social Security Design
N° 3/00	Emanuele Baldacci Luca Inglese	Le caratteristiche socio economiche dei pensionati in Italia. Analisi della distribuzione dei redditi da pensione (only available in the Italian version)
N° 2/00	Pier Marco Ferraresi Elsa Fornero	Social Security Transition in Italy: Costs, Distorsions and (some) Possible Correction
N° 1/00	Guido Menzio	Opting Out of Social Security over the Life Cycle